math MAMMOTH

Integers Worksheets

- **I**ntegers in real life
- **A**dd, subtract, multiply, and divide integers
- **N**umber lines and patterns
- **C**oordinate grid
- **I**ncludes two fact sheets

Green Series

By Maria Miller

Contents

Introduction

Math Mammoth Integers Worksheets Collection includes all integers-related worksheets from Math Mammoth Grade 6 and Grade 7 worksheets collections, here in one handy package for those who would need them.

The grade-level worksheets packages were originally created for and in collaboration with SpiderSmart, Inc. tutoring company.

These sheets cover integers and their operations in a great variety of ways. The collection starts with worksheets about the integer concept, how they are used in life, and absolute value.

Addition and subtraction are connected with temperature rise and fall, with number line models, and “+” and “-” model. One worksheet is about exploring patterns in subtraction so the student can get an intuitive feel of this operation.

Coordinate grid problems are a nice application of integer operations and connect them with geometry.

Students will also solve a few easy equations in many of the worksheets.

The collection ends in two fact sheets that sum up the four basic operations with integers. These fact sheets not only explain but also briefly justify the rules for integer operations.

I hope the problems will fit your needs.

I wish you success with math teaching!

Maria Miller, the author

Integers

1. Represent each situation with an integer.

a. Joe owes $48.

b. Jack earned $560.

c. A diver went 50 ft below sea level.

d. Temperature is 14 degrees Celsius below zero.

e. The team reached the altitude of 5,600 meters.

2. Compare. Write < or > between the numbers.

a. 8 ☐ -9 **b.** -2 ☐ 2 **c.** 0 ☐ -7 **d.** -4 ☐ 3 **e.** -5 ☐ -9

f. -10 ☐ -15 **g.** -28 ☐ 0 **h.** 13 ☐ -13 **i.** -1 ☐ -11 **j.** -1 ☐ 1

3. Order the numbers from smallest to greatest.

a. 4 -5 0 -6	**b.** -1 2 -5 1	**c.** -100 -8 -4 -7

4. Find the difference between...

a. -2°C and 1°C

b. -5°C and 4°C

c. -10°C and -8°C

d. -20°C and -5°C

e. -11°C and 15°C

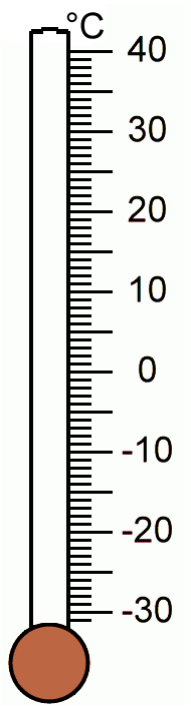
f. 6°C and -16°C

g. -9°C and - 21°C

h. 3°C and 0°C

i. 7°C and -7°C

j. -1°C and 11°C



5. Temperature changes. Find the new temperature.

before	12°C	4°C	-3°C	-9°C	-11°C	-5°C
change	drops 15°C	drops 8°C	drops 10°C	rises 10°C	rises 6°C	rises 7°C
now						

6. True or false statements? If the statement is FALSE, *explain why*.

a. Every integer is either positive or negative.

b. If you add numbers that are opposites, you get 1.

c. Any negative number is smaller than any positive number.

d. Zero is negative.

e. The opposite of -5 is 5.

f. Every integer has an opposite.

Integers

1. Write a positive or negative integer for each situation.

a. Mary owes $250.

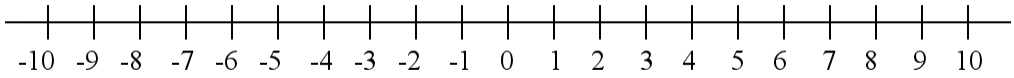
b. The diver went to the depth of 120 ft.

c. John earned $115.

d. A loss of 12 yd.

e. The temperature was 9 degrees below zero Fahrenheit.

f. 185 m above sea level



2. Compare integers.

a. 2 ☐ -2 **b.** 16 ☐ 13 **c.** 16 ☐ -14 **d.** -8 ☐ 2 **e.** -2 ☐ 8

f. -20 ☐ -25 **g.** -12 ☐ -31 **h.** 5 ☐ -7 **i.** 0 ☐ -11 **j.** -10 ☐ -1

3. Which of the integers that you see in the box below fulfill the following conditions?

$^{-}4$	$^{-}3$	$^{-}1$ 0
4	7	$^{-}5$ $^{-}8$
1	3	10

a. $x \geq {}^{-}2$

b. $^{-}3 < x < 3$

c. $x < {}^{-}4$

d. $x > {}^{-}1$

e. $^{-}3 \leq x \leq 3$

f. $x \leq 4$

4. Find a number that is....

a. 5 more than $^{-}1$

b. 6 less than 4

c. 2 less than -5

d. 4 less than zero

e. 3 more than $^{-}9$

f. 10 more than -5

g. 20 less than -20

h. 100 more than $^{-}90$

i. 1,000 less than 100

5. Continue the number sequences for six more numbers.

a. -80, -75, -70, ...

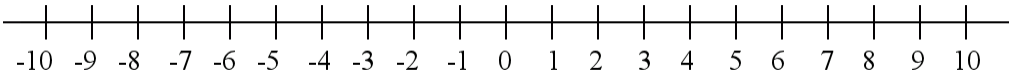
b. -23, -20, -17,...

c. 6, 3, 0, ...

6. Order the numbers from the smallest to the greatest.

a. 21 -12 12 0	**b.** -5 2 -4 3	**c.** -1 -7 3 -13
d. -5 -15 5 -9	**e.** 0 6 -10 10	**f.** -9 3 -3 -4

Adding Integers



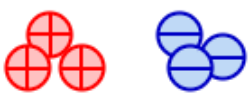
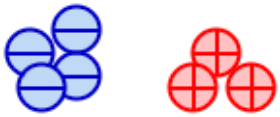
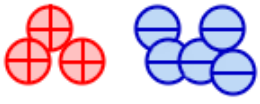
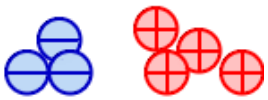
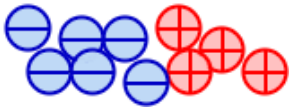
1. Add integers with same sign.

a.	**b.**	**c.**	**d.**
5 + 7 = _____	$^{-}4 + {}^{-}4$ = _____	$^{-}13 + {}^{-}31$ = _____	$^{-}20 + {}^{-}300 + {}^{-}200$ = _____
$^{-}5 + {}^{-}7$ = _____	$^{-}20 + {}^{-}10$ = _____	12 + 16 = _____	$^{-}45 + {}^{-}5 + {}^{-}10$ = _____

2. Find what is missing from the equations.

a. $^{-}3$ + _____ = $^{-}10$ **b.** $^{-}46$ + _____ = $^{-}98$ **c.** $^{-}57$ + _____ = $^{-}73$

3. Write an addition sentence.

 **a.**	 **b.**	 **c.**	 **d.**
 **e.**	 **f.**	 **g.**	 **h.**

4. Add integers with different signs.

a.	**b.**	**c.**	**d.**
5 + $^{-}4$ = _____	4 + $^{-}4$ = _____	$^{-}11 + 20$ = _____	8 + $^{-}3 + {}^{-}2$ = _____
$^{-}5 + 8$ = _____	$^{-}9 + 9$ = _____	2 + $^{-}6$ = _____	$^{-}4 + 5 + 1$= _____
$^{-}5 + 10$ = _____	$^{-}30 + 20$ = _____	$^{-}9 + 3$ = _____	$^{-}5 + 16 + {}^{-}10$ =_____

5. Make addition sentences. <u>Use one positive and one negative integer.</u>

a.	**b.**	**c.**	**d.**
_____ + _____ = $^{-}1$	_____ + _____ = 0	_____ + _____ = $^{-}3$	_____ + _____ = 2
_____ + _____ = $^{-}1$	_____ + _____ = 0	_____ + _____ = $^{-}3$	_____ + _____ = 2
_____ + _____ = $^{-}1$	_____ + _____ = 0	_____ + _____ = $^{-}3$	_____ + _____ = 2

6. Find what is missing from the equations.

a. 2 + _____ = $^{-}5$ **b.** $^{-}4$ + _____ = 8 **c.** $^{-}5$ + _____ = 1

Add and Subtract Integers

1. The temperature is rising so we add a positive number.

a. The temperature was -3°C. It rose 4 degrees. Now it is _____.

b. The temperature was -5°C. It rose 2 degrees. Now it is _____.

c. The temperature was -9°C. It rose 4 degrees. Now it is _____.

d. The temperature was -1°C. It rose 7 degrees. Now it is _____.

e. The temperature was -2°C. It rose 5 degrees. Now it is _____.

f. The temperature was -10°C. It rose 3 degrees. Now it is _____.

Addition:

$^{-}3 + 4 = 1$

2. The temperature is dropping so we subtract a positive number.

a. The temperature was 3°C. It dropped 4 degrees. Now it is _____.

b. The temperature was 7°C. It dropped 10 degrees. Now it is _____.

c. The temperature was 5°C. It dropped 8 degrees. Now it is _____.

d. The temperature was -1°C. It dropped 5 degrees. Now it is _____.

e. The temperature was -7°C. It dropped 7 degrees. Now it is _____.

f. The temperature was -10°C. It dropped 2 degrees. Now it is _____.

Subtraction:

3. Solve the problems, and observe the patterns.

a.	**b.**	**c.**	**d.**
$5 - 4 =$ _____	$^{-}4 - 0 =$ _____	$^{-}3 + 0 =$ _____	$^{-}2 + 2 =$ _____
$5 - 5 =$ _____	$^{-}4 - 1 =$ _____	$^{-}3 + 1 =$ _____	$^{-}2 + 3 =$ _____
$5 - 6 =$ _____	$^{-}4 - 2 =$ _____	$^{-}3 + 2 =$ _____	$^{-}2 + 4 =$ _____
$5 - 7 =$ _____	$^{-}4 - 3 =$ _____	$^{-}3 + 3 =$ _____	$^{-}2 + 5 =$ _____
$5 - 8 =$ _____	$^{-}4 - 4 =$ _____	$^{-}3 + 4 =$ _____	$^{-}2 + 6 =$ _____

4. Match the equations with the situations, and complete the missing parts.

a. A diver was at the depth of 20 ft. Then he rose 15 ft.
Now he is at _____ ft.

b. John had $15. He had to pay his dad $20. Now he _____.

c. John had a $15 debt. He earned $20. Now he _____.

d. A ball was dropped from 15 ft above the sea; it fell 20 ft.
Now the ball is at _____ ft.

e. The temperature was 20°C and fell 15°. Now it is _____ °C.

$15 - 20 =$ _____

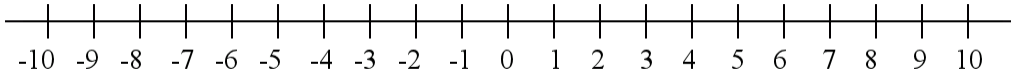
$-15 + 20 =$ _____

$-20 + 15 =$ _____

$20 - 15 =$ _____

$15 - 20 =$ _____

Number Line Jumps



1. Write an addition sentence.

Addition:

a. You are at ¯2. You jump 6 to the right. You end up at _____.

b. You are at ¯4. You jump 5 to the right. You end up at _____.

c. You are at ¯7. You jump 2 to the right. You end up at _____.

d. You are at ¯10. You jump 4 to the right. You end up at _____.

2. Write a subtraction sentence.

Subtraction:

a. You are at 2. You jump 6 to the left. You end up at _____.

b. You are at 5. You jump 10 to the left. You end up at _____.

c. You are at ¯4. You jump 4 to the left. You end up at _____.

d. You are at ¯7. You jump 3 to the left. You end up at _____.

3. Add and subtract. You can think of number line jumps.

a.	**b.**	**c.**	**d.**
5 − 7 = _____	¯5 − 5 = _____	¯3 + 6 = _____	¯20 + 20 = _____
10 − 20 = _____	¯4 − 10 = _____	¯12 + 7 = _____	¯40 + 70 = _____
14 − 18 = _____	¯8 − 12 = _____	¯100 + 50 = _____	¯10 + 4 = _____
50 − 70 = _____	¯40 − 12 = _____	¯7 + 1 = _____	¯7 + 6 = _____

4. Draw a number line from -25 to -10.

5. Draw jumps on the number line above that illustrate the additions:

a. ¯21 + 5 = _____ **b.** ¯24 + 10 = _____ **c.** ¯16 − 8 = _____ **d.** ¯11 − 2 = _____

6. Find what is missing from the equations.

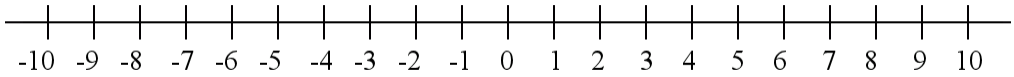
a. 4 − _____ = ¯4 **c.** ¯8 + _____ = ¯4 **e.** 4 − ___= ¯6 **g.** ¯3 + _____ = 5

b. 3 − _____ = ¯5 **d.** ¯6 + _____ =¯1 **f.** 3 − ___= ¯8 **h.** ¯5 + _____ = 9

Opposites and Absolute Value

1. Fill in the chart.

Integer	−5	8			−19			0
Opposite			−6	11				
Absolute value						4		



2. Mark with symbols. Then evaluate the expression you have written.

a. opposite of 7

b. absolute value of ¯7

c. opposite of absolute value of 7

d. opposite of absolute value of ¯7

3. What is the number whose opposite is...

a. 12

b. ¯6

c. one more than ¯2

d. two less than 5

e. five more than ¯6

f. two more than ¯1

4. What are the two numbers whose absolute value is...

a. 10 **b.** 55 **c.** one less than 4 **d.** seven more than −2

5. Evaluate each expression.

a. −(−9)

b. −(+9)

c. +(−9)

d. −|−9|

e. |−12|

f. |20|

g. − |4|

h. |6 − 2|

i. −(6 + 5)

j. |5 + 6|

k. −(−6)

l. 10 − |−4|

6. Compare the integers. Under each pair, write the difference of the two.

a. −6 ☐ 3 **b.** 5 ☐ −2 **c.** −8 ☐ −6 **d.** 10 ☐ 3 **e.** 4 ☐ −7

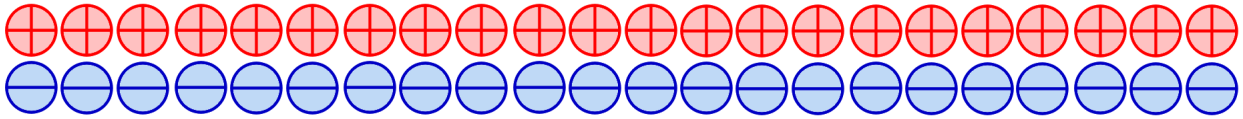
Difference:

7. Write the absolute values of the numbers in the previous problem, compare them, and write under them the difference of their absolute values.

a. 6 ☐ 3 **b.** ☐ **c.** ☐ **d.** ☐ **e.** ☐

Difference: 3

Add Integers 1



1. Add. You can use the minus and plus points above to help.

a. $^{-}1 + 6$ **b.** $^{-}4 + {}^{-}2$ **c.** $4 + {}^{-}5$ **d.** $4 + {}^{-}3$

e. $^{-}3 + 1$ **f.** $^{-}3 + {}^{-}3$ **g.** $^{-}10 + 11$ **h.** $^{-}5 + {}^{-}2$

2. Calculate.

a. 6 + (-2) + (-3) **b.** (-3) + (-1) + (-10) **c.** 2 + (-4) + 5

3. Write the sum of the numbers.

a. 6 and -8 **b.** -4 and -5 **c.** -4 and 8

d. 7, -8, and -1 **e.** 3, -5, and 1 **f.** -10, -5, 3, and 6

4. Find the value of the expression $x + y$, when

a. $x = {}^{-}5$ and $y = 6$ **b.** $x = {}^{-}5$ and $y = {}^{-}6$ **c.** $x = 5$ and $y = {}^{-}6$

5. Mark is using a credit card for his purchases so he can buy stuff even when he doesn't really have the money. Write an addition sentence. Let each spending be a negative integer.
In the end, is Mark owing money or not, and how much does he have or owe?

Started out with $20. Spent $12. Spent $15. Spent $12. Earned $25. Spent $10. Earned $50.

6. Continue the patterns.

a.	**b.**	**c.**	**d.**
6 + (-3) =	-10 + 6 =	(-3) + (-4) =	(-90) + 10 =
6 + (-4) =	-10 + 7 =	(-3) + (-3) =	(-90) + 15 =
6 + (-5) =	-10 + 8 =	(-3) + (-2) =	(-90) + 20 =
6 + (-9) =	-10 + 12 =	(-3) + 2 =	(-90) + 40 =

Add Integers 2

Add two integers with same sign:	**Add two integers with different sign:**
Find the sum of their absolute values. The answer has the _____ sign as the addends.	Find the _____ of their absolute values. The answer has the same sign as _____

1. Add these integers, which have the same sign.

a. ¯15 + ¯17 **b.** 130 + 100 + 20 **c.** ¯130 + ¯100 + ¯20

2. Add these integers, which have different signs.

a.	**b.**	**c.**
14 + ¯4 =	¯18 + 8 + 9 =	4 + ¯9 + ¯3 + 2 =
¯11 + 11 =	15 + ¯6 + ¯5 =	¯20 + 15 + 10 + ¯45=
¯15 + 20 =	¯6 + 7 + ¯9 =	¯7 + 6 + ¯1 + ¯9=

3. Find the value of the expression a + b + 20 when

a. *a* = -20 and *b* = 15. **b.** *a* = -157 and *b* = -250. **c.** *a* = 452 and *b* = -741.

4. Solve the equations.

a. 3 + x = (-6) **b.** (-7) + y = (-2) **c.** (-4) + *z* = 6

d. (-3) + x = (-10) **e.** (-10) + y + (-9) = (-20) **f.** *z* + (-157) = (-729)

5. Adding up to zero! To one!

a. ____ + 5 = 0	**b.** 4 + ____ + 2 = 0	**c.** _____ + 3 = 1	**d.** (-4) + _____ + 5 = 1
(-9) + ____ = 0	(-8) + ____ + ____ = 0	11 + _____ = 1	(-7) + ____ + ____ = 1
145 + _____ = 0	450 + ____ + ____ = 0	(-150) + _____ = 1	6 + _____ + ____ = 1

6. Evaluate the expressions if *a* = ¯165 and *b* = ¯89.

a. *a* + *b* + 900 **b.** *a* + *a* + *a* + *b* + *b* **c.** *a* + ¯51 + *b* + 218

Explore Subtraction

1. Fill in the tables using logical reasoning!

a.

x	5 − x
2	5 − 2 = 3
3	5 − 3 = 2
4	
5	
6	
7	
8	
9	

b.

y	100 − y
60	
70	
80	
90	
100	
110	
120	
130	

c.

x	500 − x
300	
350	
400	
450	
500	
550	
600	
650	

2. What about subtracting a *negative* number? Continue the patterns <u>logically</u>.

a.

x	6 − x
3	6 − 3 = 3
2	6 − 2 = 4
1	
0	
-1	
-2	
-3	
-4	

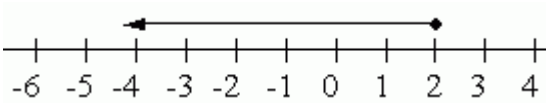
b.

x	10 − x
4	10 − 4 = 6
6	
8	
10	
12	
14	
16	
18	

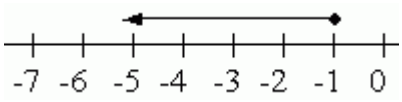
c.

x	30 − x
10	30 − 10 = 20
20	30 − 20 =
30	
40	
50	
60	
70	
80	

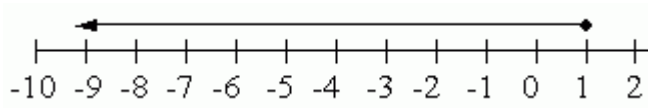
3. Subtracting a whole number is like a jump or arrow that moves many steps towards the left. Write a subtraction sentence for each picture.



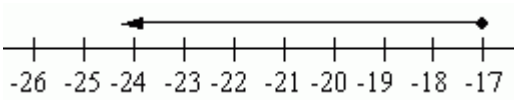
a. 2 − 6 =



b.



c.



d.

4. Think you got it? Try these!

a. 2 − 7 =	**b.** 2 − ¯3 =	**c.** ¯3 − ¯5 =
5 − 8 =	1 − ¯20 =	¯7 − ¯10 =
20 − 50 =	40 − ¯15 =	¯15 − ¯15 =
127 − 459 =	492 − ¯293 =	¯928 − ¯823 =

Subtract Integers

1. You can ALWAYS change any subtraction into addition: **Instead of subtracting a number, add its o_____.**	
a. 5 − 2 = 5 + (−2) =	**c.** 7 − (−3) = 7 + _____ =
b. (−4) − 11 = −4 + _____ =	**d.** (−6) − (−10) = (−6) + _____ =

2. Time for practice! Subtract.

a. (−3) − 7 = (−7) − 13 = (−21) − 51 =	**b.** 3 − (−7) = 7 − (−13) = 21 − (−51) =	**c.** (−3) − (−7) = (−7) − (−13) = (−21) − (−51) =

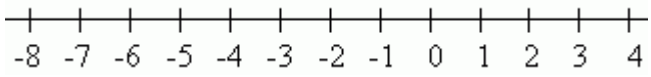
3. Write the expression for the difference of these numbers, and calculate:

a. 4 and ¯2 **b.** ¯9 and 11 **c.** ¯5 and ¯20 **d.** ¯8 and ¯100

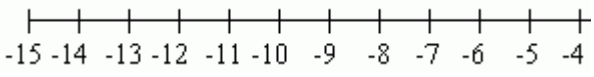
4. Consider the numbers (−6) and 4. Write down the expression and calculate

a. their difference **b.** their sum **c.** the sum of their absolute values

5. Draw an arrow on the number line to illustrate the subtraction and solve.



a. 3 − 7



b. −6 − 7

6. The temperature falls, so we subtract. T refers to *temperature*.

Initial T	2°C	3°C	-1°C		-2°C	5°C	
Drop	3°C	4°C	2°C	4°C			2°C
Final T				-1°C	-4°C	-3°C	-10°C
Subtraction sentence	2 − 3 = −1						

7. Subtract many times!

a. (−3) − 7 − 5 = (−8) − (−10) − 12 = (−30) − 50 − 100 =	**b.** 1 − 2 − 3 − 4 − 5 = 9 − (−8) − (−7) = − (−11) − (−12) − (−13) =

Add / Subtract Roundup

1. Write “rules” or mnemonics that explain and help you remember how to add and subtract integers.

a. Adding two negative numbers:	**b. Adding a negative and a positive number:**
c. Subtracting a positive number:	**d. Subtracting a negative number:**

2. Time for practice! Add and subtract.

a. 3 + (− 7) =	**b.** 40 − (−70) =	**c.** (−3) + (−17) =
5 − (−13) =	(−7) + (−13) =	(−2) − (−6) =
(−50) + 20 =	28 + (−2) =	2 − (−5) =

3. Let $a = (-4)$ and $b = 7$. Find the value of the expressions.

a. $a - b - 10$ **b.** $100 - (a + b)$ **c.** $100 - (a - b)$

4. Calculate.

a. (−7) + 8 − 10 **b.** 10 + (−8) − 5 **c.** (−14) − 8 + (−2)

5. Solve the equations.

a. x + 9 = 8	**b.** 10 − a = 11	**c.** z − 5 = (−1)
y + 5 = 1	50 − b = 55	w − 10 = (−6)

6. This is addition only! What is the quickest way?

a. 3 + (−7) + 9 + (−4) + 8 + (−10) + 11 =

b. 5 + (−15) + (−20) + 15 + (−30) + 25 + (−50) + (−5) =

7. Calculate.

a. (−5) + 8 − 4 + (−2) **b.** 123 + (−124) − 122 − (−121)

8. Let's not forget absolute values. Simplify.

a. −|−9| −(− 8)

b. −|8 + (−2)| −(4 − 8)

c. −|1 + 12| +(− 8)

d. −|−6 −(−2)| −(−4 + 8)

Multiply Integers

Multiplication by a whole number is repeated addition:

$4 \times \bar{3} = \bar{3} + \bar{3} + \bar{3} + \bar{3} =$ _____

1. Multiply integers.

a.	**b.**	**c.**	**d.**
$7 \times \bar{3} =$ _____	$6(-4) =$ _____	$0 \times \bar{3}1 =$ _____	$8 \times 0 \times \bar{3}1 =$ _____
$2 \times \bar{5} =$ _____	$10(-10) =$ _____	$2 \times \bar{4}5 =$ _____	$4 \times 10 \times \bar{7} =$ _____
$5 \times \bar{8} =$ _____	$100(-1) =$ _____	$8 \times \bar{5}0 =$ _____	$2 \times 5 \times \bar{8} =$ _____

2. Find what is missing from the equations.

a. $3 \times$ _____ $= \bar{3}0$ **b.** $4 \times$ _____ $= \bar{1}00$ **c.** $6 \times$ _____ $= \bar{7}2$ **d.** $7 \times$ _____ $= \bar{7}70$

3. These points form a rectangle: (2,1), (-1,1), (2, -1), (-1,-1).

a. Plot the points and draw the rectangle.

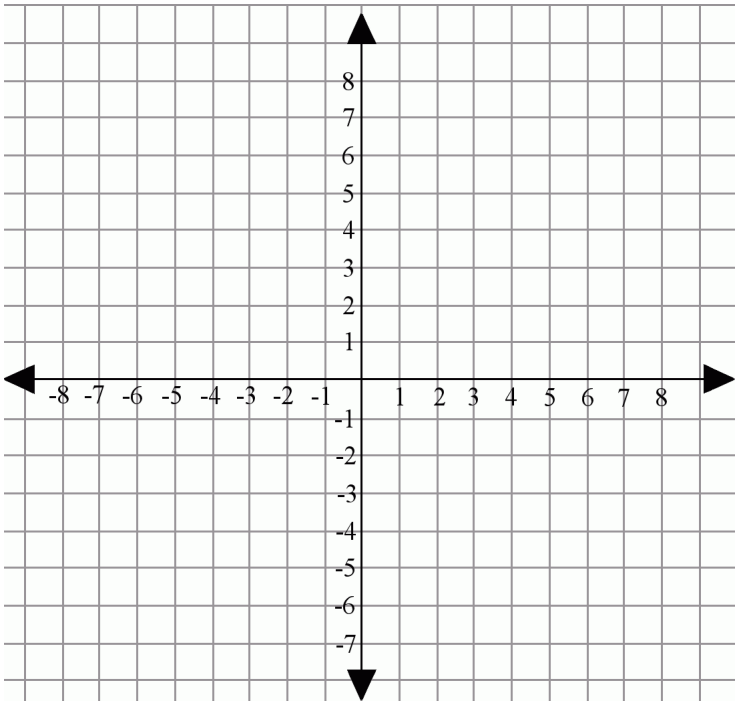
b. Multiply all the coordinates by 2 to get four new points and rectangle 2.

c. Multiply all the original coordinates by 3 to get four new points and rectangle 3.

d. Multiply all the original coordinates by 4 to get four new points and rectangle 4.

e. Compare the side lengths of the rectangles. What can you notice?

f. Write here the areas of the rectangles.



g. What is the ratio (area of rectangle 1) to (area of rectangle 2)?

h. What is the ratio (area of rectangle 1) to (area of rectangle 3)?

i. What is the ratio (area of rectangle 1) to (area of rectangle 4)?

Multiply Integers

1. Complete the little multiplication 'rules'.

2. Write each addition as a multiplication, and solve.

a. $^{-}2 + {}^{-}2 + {}^{-}2 + {}^{-}2$ **b.** $^{-}11 + {}^{-}11$ **c.** $^{-}100 + {}^{-}100 + {}^{-}100$

3. Multiply.

a. $4 \times {}^{-}2$ **b.** $^{-}2 \times 11$ **c.** $^{-}3 \times {}^{-}10$ **d.** $^{-}1 \times {}^{-}100$

e. $8\ (\ {}^{-}2)$ **f.** $^{-}2\ (^{-}7)$ **g.** $(^{-}3)10$ **h.** $(\ {}^{-}1) \times (^{-}1)$

4. Continue and fill in the tables!

a.

x	2x
3	2(3) = 6
2	2(2) =
1	2(1) =
0	2(0) =
−1	2(−1) =
−2	
−3	
−4	

b.

y	(−3)y
3	(−3)(3) = −9
2	(−3)(2) =
1	(−3)(1) =
0	(−3)(0) =
−1	(−3)(−1) =
−2	
−3	
−4	

c.

a	5a + 1
2	5(2) + 1 = 11
1	5(1) + 1 = 6
0	
−1	
−2	
−3	
−4	
−5	

5. Calculate.

a. $4 \times (-10) + 80$ **b.** $(-5) \times 11 - 40$ **c.** $45 + (-3) \times (-10)$

6. Evaluate the expressions when $x = (-10)$ and $y = 2$.

a. xy **b.** $-xy$ **c.** $100 - xy$ **d.** $xy - 20$

7. Multiply many numbers!

a. $(-10) \times 5 \times (-2)$ **b.** $4 \times (-4) \times 0 \times (-9)$ **c.** $100 \times (-1) \times (-3)$

d. $(-3) \times (-2) \times (-5) \times (-2)$ **e.** $2 \times (-5) \times (-10) \times 5 \times (-3)$ **f.** $2 \times (-3) \times 4 \times (5)$

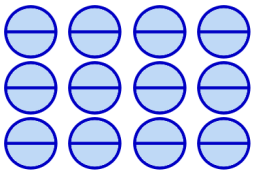
8. Solve the equations.

a. $(-10)y = 100$ **b.** $4a = -36$ **c.** $z(-7) = -49$

d. $-3w = (-48)$ **e.** $-5x = -55$ **f.** $8z = -64$

Divide Integers

1. Divide the 12 negatives into four equal groups and write a division sentence.



2. For each division, there is a multiplication sentence. Write the missing calculation.

a. $(-6) \div (-3) = \_\_\_$ $2 \times (-3) = -6$	**b.** $21 \div (-7) = \_\_\_$ $\_\_\_ \times (-7) = \_\_\_$	**c.** $4 \times (-5) = \_\_\_$
d. $-24 \div 6 = \_\_\_$	**e.** $(-80) \div (-10) = \_\_\_$	**f.** $(-7) \times (-5) = \_\_\_$

3. Divide.

a. $4 \div {}^{-}2$ **b.** ${}^{-}30 \div 3$ **c.** ${}^{-}18 \div {}^{-}6$ **d.** ${}^{-}100 \div {}^{-}1$

e. $80 \div (-2)$ **f.** $(-2) \div (-2)$ **g.** $(-2) \div (-1)$ **h.** $1 \div (-1)$

4. Consider the numbers ${}^{-}20$ and ${}^{-}2$. Write down and calculate their

a. sum **b.** difference **c.** product **d.** quotient

5. Calculate and compare!

a. $2 \div (-2) =$ $(-2) \div 2 =$ $2 \div 2 =$ $(-2) \div (-2) =$	**b.** $7 \div (-1) =$ $(-7) \div 1 =$ $7 \div 1 =$ $(-7) \div (-1) =$	**c.** $752 \div (-8) =$ $(-752) \div 8 =$ $752 \div 8 =$ $(-752) \div (-8) =$

6. Calculate.

a. $(-10) \div 5 \times (-2)$ **b.** $40 \div (-4) + 70 \div (-7)$ **c.** $(50 - 80) \div (-3)$

7. Expressions tend to be easier to read when division is written using the fraction line.

a. $\dfrac{6 + (-10)}{(-4)}$ **b.** $\dfrac{8\,(-20)}{10} - 10$ **c.** $\dfrac{(-60)}{4} + \dfrac{60}{(-4)}$

8. Solve the equations.

a. $y \div (-10) = 100$ **b.** $36 \div a = -4$ **c.** $z + (-7) = -11$

d. $5w = (-50)$ **e.** $-63 \div x = -9$ **f.** $m \div 4 = -12$

Integer Problems

1. Write a number sentence using integers.

a. Romulus, king of Rome, came into power in -753 (or 753 BC) and reigned for 37 years. When did his reign end?

b. The temperature was −4°C, but then increased by seven degrees. What is the temperature now?

c. The temperature was −6°C, but then it got six degrees colder. What is the temperature now?

d. The temperature was 1°C, but then it dropped by five degrees. What is the temperature now?

e. On his credit card account, Jack had $15. Then he spent $50. What is his account balance now?

f. These are Jack's debts to his dad that he has made in different times: $45, $24, $20, $50 and $10. What is his total debt?

g. A submarine was at the depth of 80 ft. Then it rose 20 ft. Then it descended 40 ft. Then it descended 25 ft more. Lastly, it rose 60 ft. What is its current depth?

2. Find the value of the expressions.

a. $76 + (-428)$

b. $-40 - 4 - 4 - 4 - 4 - 4$

c. $(-82) \times (-45)$

d. $(-8) \times (-4) \times (-1) \times 2 \times 5$

e. $|3 - 5|$

f. $-(-2 + 1)$

g. $\dfrac{|-8| + |20| + |-14|}{7}$

h. $-(40 + (-4))$

i. $\dfrac{(-8) + (-20) + (-16)}{(-4)}$

3. Find the average of these temperatures: 2°C, 12°C, −6°C, 7°C, −5°C, 5°C, −1°C.

4. True or false? If false, provide an example that shows the sentence is not true.

a. The opposite of 4 is the same as the opposite of −4.

b. The sum of the absolute values of −3 and 3 is six.

c. The integers that are less than 2 have absolute values less than 2.

d. The opposite of 9 has the same absolute value as 9.

e. When you multiply a number by −1, you get its opposite.

Powers with a Negative Base

1. Write each power as a repeated multiplication, and solve.

a. $(-2)^3 = (-2) \times (-2) \times (-2) =$

b. $(-10)^2$

c. $(-4)^2$

d. $(-10)^3$

2. Continue the patterns and solve. Then compare!

a.	**b.**	**c.**	**d.**
$10^1 =$	$(-10)^1 =$	$2^1 =$	$(-2)^1 =$
$10^2 =$	$(-10)^2 =$	$2^2 =$	$(-2)^2 =$
$10^3 =$	$(-10)^3 =$	$2^3 =$	$(-2)^3 =$
$10^4 =$	$(-10)^4 =$	$2^4 =$	$(-2)^4 =$
$10^5 =$	$(-10)^5 =$	$2^5 =$	$(-2)^5 =$
$10^6 =$	$(-10)^6 =$	$2^6 =$	$(-2)^6 =$
$10^7 =$	$(-10)^7 =$	$2^7 =$	$(-2)^7 =$
$10^8 =$	$(-10)^8 =$	$2^8 =$	$(-2)^8 =$

3. Solve.

a. $(-4)^3$ **b.** $(-1)^6$ **c.** $(-5)^2$ **d.** $(-5)^3$

e. $(-2)^5$ **f.** 100^2 **g.** $(-1)^7$ **h.** $(-3)^4$

4. Calculate.

a. $4(-10) + (-10)^4$ **b.** $(-5 + 11)^2$ **c.** $(-2 - 10)^3$

d. $(-2)^5 + 2^5$ **e.** $(-3)^4 + 3^4$ **f.** $\dfrac{(-2)^5}{(-2)^3}$

5. Find the value of these expressions when $x = (-2)$ and $y = 5$.

a. $x^5 - 6y$ **b.** $7x^2 + y^2$ **c.** $3x^3 - 6y^2$ **d.** x^2y

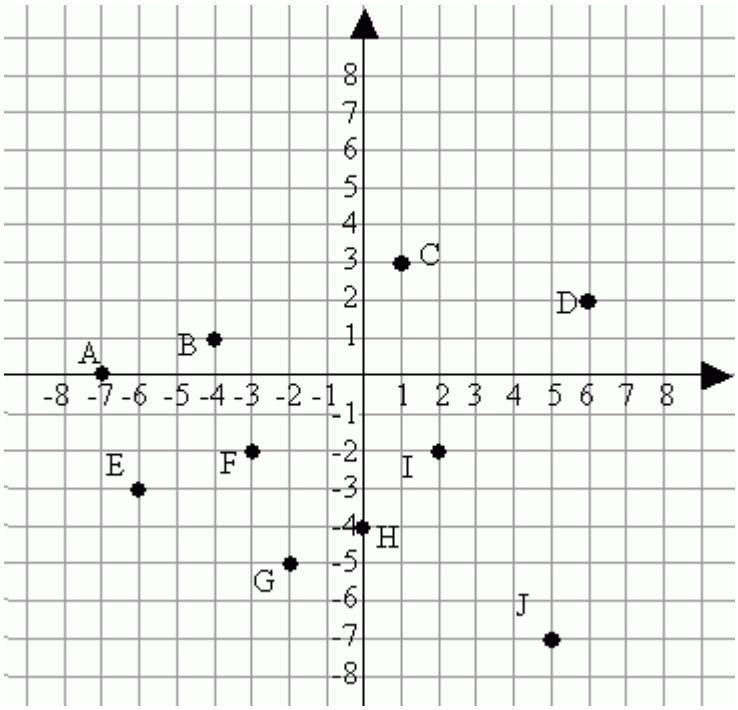
6. What number fits in place of x ?

a. $(-10)^x = 100$ **b.** $(-5)^x = -3125$ **c.** $4^x = 64$ **d.** $(-1)^x = 1$

Coordinates

1. Write the coordinates of the points marked on the coordinate grid.

A
B
C
D
E
F
G
H
I
J



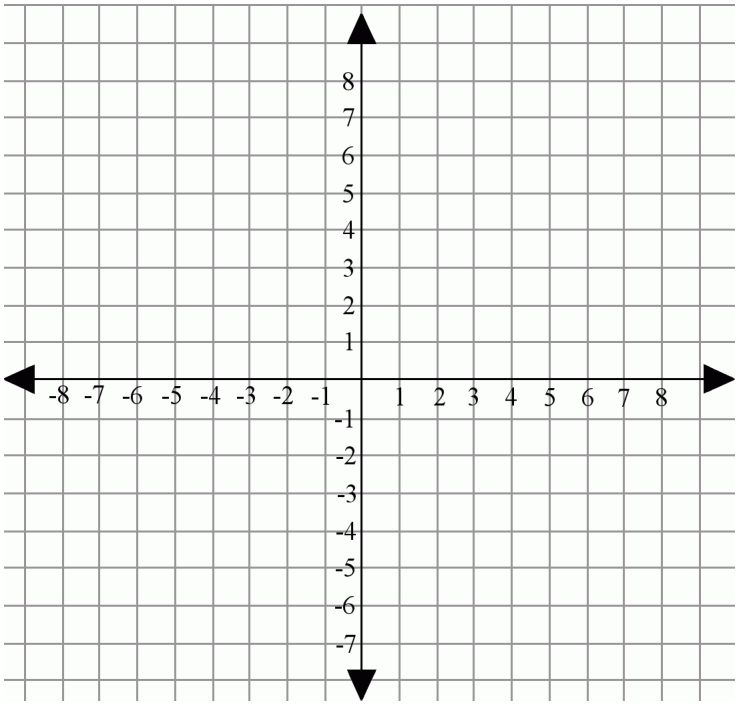
2. Plot the following sets of points on the empty grid below.
Which figures are formed?

a. (-2, 4), (2,4), (-2,0), (2,0)

b. (-8,-8), (-5,-5), (-2,-5), (-5,-8)

c. (2, -7), (3, -4), (5,-4), (8,-7)

d. (-8,0), (-4,8), (0,0)



3. Find the new point based on the directions. (You don't have to plot the points; just write their coordinates.)

Point	Direction	New point
(-2,4)	two up, two left	
(1,1)	3 down, 7 left	
(0,-3)	four up, five left	
(2,-5)	6 up, 6 left	
(0,0)	2 down, 1 right	
(5,8)	14 down, 7 left	
(-5,-6)	10 up, 10 right	
(-2,-2)	5 up, 7 right	

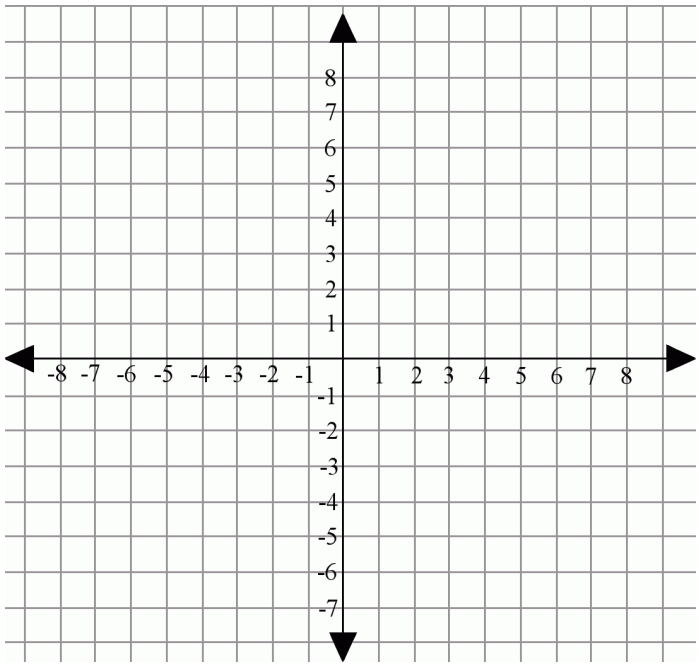
Movements in Coordinate Grid

1. Draw a rectangle 5 units wide and 3 units tall whose lowest left corner is at (-7,4).

   Write here the coordinates of its vertices.

2. Now move your rectangle 7 units down. Write the coordinates of its vertices now.

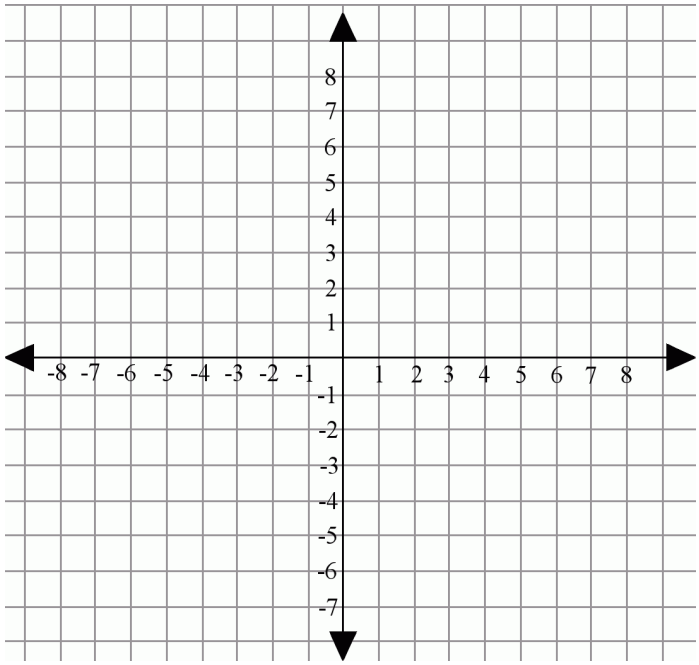
3. Now the rectangle moves 10 units to the right and 10 units up from its position in problem (2). Find the coordinates of its vertices.



4. This time, ***calculate*** first, do not draw. The table lists vertices of a triangle, and how they are moved. Find the coordinates of the new vertices.

Vertices	Direction	New vertices
(3,-4)	two up, three to the left	
(5,-5)		
(4,0)		

Check by drawing.



5. A parallelogram was moved three units up and three units to the right, so that the new coordinates for its vertices are (0,0), (4,0), (5,2), and (1,2). What were the coordinates of the vertices originally?

6. The point (-6,-6) is moved two units up and one to the right repeatedly 5 times. Where will it end up after that?
   What if the point (10,30) was moved in a similar way 100 times? Where would it end up?

Functions in Coordinate Grid 1

1. Figure out how the y-coordinate relates to the x-coordinate. Plot the points and connect them.

a.

x	y
-1	1
0	2
1	3
2	
3	
4	

b.

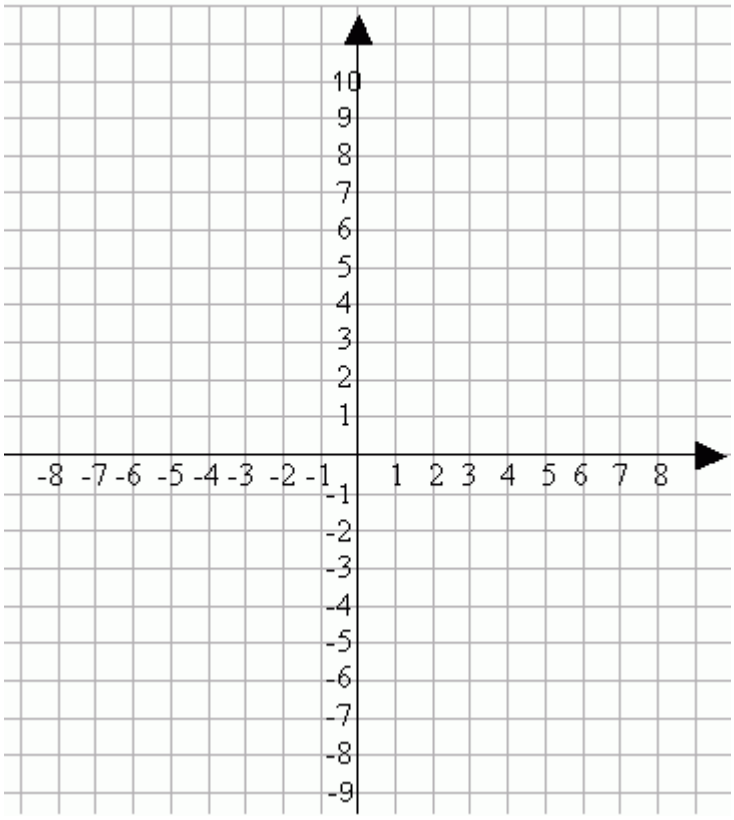
x	y
-6	-3
-4	-1
-3	
0	
3	
5	8

c.

x	y
-5	-6
-4	
-1	
0	-1
5	
6	5

d.

x	y
-3	-6
-1	
0	0
1	
2	
4	8



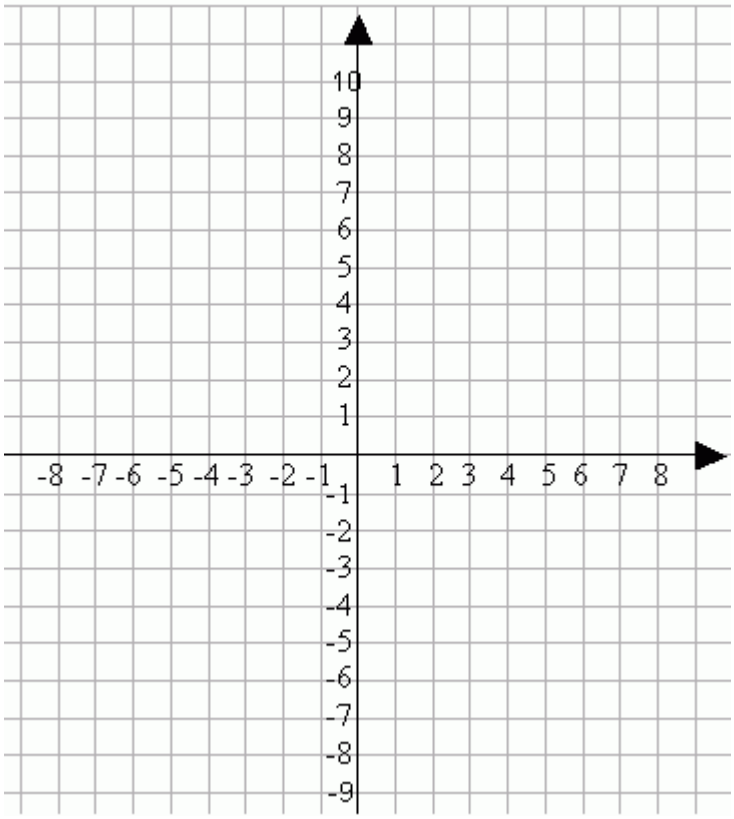
2. Use the given rule to find the y-coordinates. Plot the points and connect them.

a.

x	y = 2x + 1
-5	
-3	
-1	
1	
3	
5	

b.

x	y = 2 x − 2
-3	
-1	
0	
1	
3	
5	



Functions in Coordinate Grid 2

1. Figure out how the y-coordinate relates to the x-coordinate. Plot the points and connect them.

a.

x	y
-4	
-3	
-2	-2
-1	0
0	
1	4
2	
3	
4	10

b.

x	y
-6	-3
-5	-2.5
-4	
-3	
-2	
3	
4	
5	
6	3

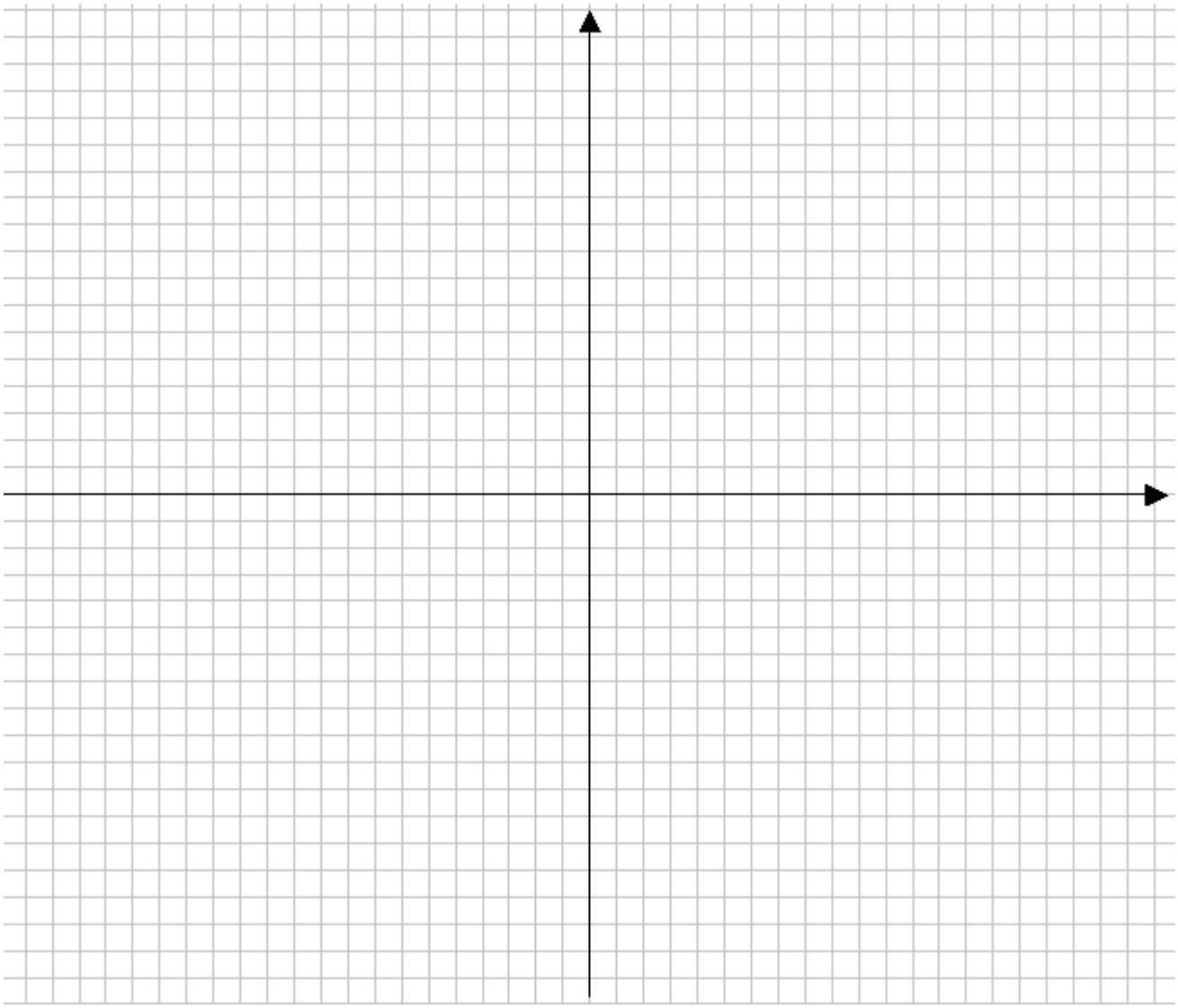
2. Use the given rule to find the y-coordinates. Plot the points and connect them.

a.

x	y = 3x + 7
-5	
-4	
-3	
-2	
-1	
0	
1	
2	
3	

b.

x	y = 1/2x + 5
-4	
-3	
-2	
-1	
0	
1	
2	
3	
4	



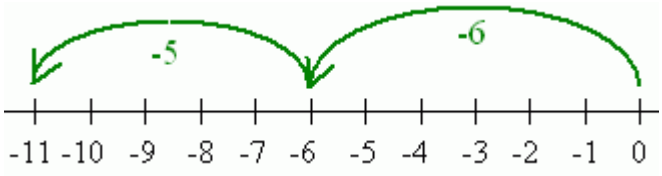
Integers Reminder Sheet 1

Adding negative integers.

(−1) + (−3) = (−4)

(−6) + (−12) + (−3) + (−10) + (−5) = −36
debt debt debt debt debt = lots of debt

You jump on the number line towards the negative (left), more and more.



(−6) + (−5) = −11

Just add the absolute values, and put the negative sign in front.

Subtracting a positive integer.

2 − 1 = 1	(−4) − 0 = −4
2 − 2 = 0	(−4) − 1 = −5
2 − 3 = −1	(−4) − 2 = −6
2 − 4 = −2	(−4) − 3 = −7
etc.	etc.

It is like the temperature dropping, or money being subtracted from a bank account. Subtracting a positive integer just means more debt.

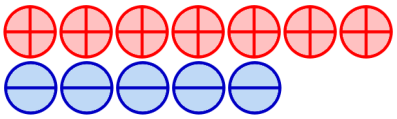
On a number line, subtracting 7 means jumping 7 steps towards the left.

5 − 8 is also the same as 5 + (−8). In other words, you can change subtracting a number into addition of the opposite number.

Let's say the answer to (−2) − 6 is A and we don't know it yet. Since subtraction is the opposite operation of addition, A + 6 = −2.
A = −8 is the only number that works.

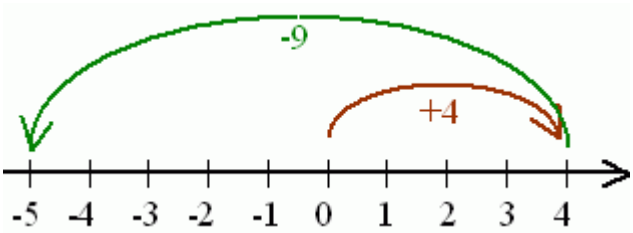
Adding integers with different signs.

7 + (−5) = ___



Some of the positives & negatives cancel each other. The difference of the absolute values tells you how many didn't get canceled.

Jump on a number line: a positive number is a jump to the right, a negative number means a jump to the left.



4 + (−9) = −5.

Subtracting a negative integer.

2 − 2 = 0	(−7) − 2 = (−9)
2 − 1 = 1	(−7) − 1 = (−8)
2 − 0 = 2	(−7) − 0 = (−7)
2 − (−1) = 3	(−7) − (−1) = (−6)
2 − (−2) = 4	(−7) − (−2) = (−5)
etc.	etc.

When you subtract a negative integer, change it into adding the opposite of the negative number, which is of course positive:

8 − (−4) = 8 + 4 = 12.
(−10) − (−4) = (−10) + 4 = (−6)

Two negatives changes into one positive!

On a number line, subtracting (−4) means jumping 4 steps to the left - but before you jump, the extra minus makes you turn around, so you jump to the right instead.

Let's say the answer to 2 − (−3) is B and we don't know it yet. Since subtraction is the opposite operation of addition, B + (−3) = 2.
B = 5 is the only number that works.

Similarly, if (−7) − (−3) is C, then C + (−3) = (−7).
So C is (−4).

Integers Reminder Sheet 2

A positive integer times a negative integer:

Think of repeated addition here: 3 × (−2) =

(−2) + (−2) + (−2) = −6.

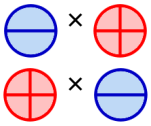
Or, 4 × (−7) = (−7) + (−7) + (−7) + (−7) = −28.

A positive integer times a negative integer:

Since you can change the order of the factors,

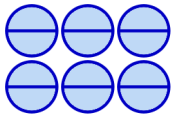
(−6) × 4 = 4 × (−6) = −24.

In general, if *m* and *n* are natural numbers, then *m* × (−*n*) is (−*n*) added repeatedly *m* times, so is negative. And (−*m*) × *n* is the same as *n* × (−*m*) and so is negative as well.



both have a negative answer

Dividing a negative integer by a positive.



Divide these negatives into three groups.

(−6) ÷ 3 = −2.

Dividing a positive integer by a negative.

What is (−15) ÷ 5 ? Let's call the answer Z. Since division and multiplication are opposite operations, Z × 5 = −15. So Z must be −3.

In general, if *m* and *n* are natural numbers, and (−m) ÷ n is B, then B × n = (−m), and B must be negative.

Dividing a negative integer by a negative.

Let's say (−21) ÷ (−7) is some number A.

It follows that A × (−7) = (−21)

Knowing the multiplication rules,
the only number that fits A is 3.

In general, if *m* and *n* are natural numbers, and (−m) ÷ (−n) is B, then B × (−n) = (−m),
and B must be positive.

A negative times a negative.

(−3) × 3 =
(−3) × 2 =
(−3) × 1 =
(−3) × 0 =
(−3) × (−1) =
(−3) × (−2) =
(−3) × (−3) =
(−3) × (−4) =

Complete the pattern on the left. Observe how the products continually increase by 3 in each step.

It follows that the *negative times negative* products in the pattern must be positive.

Another 'justification' for this rule can be seen using distributive property:

Distributive property of arithmetic states that a(b + c) = ab + ac.

So, if a = (−1), b = 3, and c = (−3), it should still hold:

(−1)(3 + (−3)) = (−1)(3) + (−1)(−3)

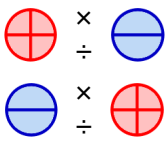
Now, since 3 + (−3) is zero, the whole left side is zero. So (−1)(3) + (−1)(−3) must be zero as well.

(−1)(3) is −3. So it follows that (−1)(−3) has to be the opposite of −3, or 3.

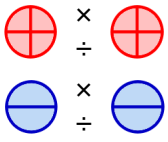
The *'negative times negative makes a positive'* rule has to do with the fact that IF we made it to be negative, then all these neat rules and properties of arithmetic wouldn't hold for negative numbers.

But mathematicians do want them to hold, since we DO want mathematics to be a very consistent system. So the convention is made that negative times negative is positive.

In a nutshell, whether you are multiplying *or* dividing:



(different signs)
yields a negative answer.



(same kind of signs)
yields a positive answer.

Math Mammoth Integers Worksheets Answer Key

Integers, p. 5

1. a. -48 b. 560 c. -50 ft d. -14 °C e. 5,600 m

2. a. > b. < c. > d. < e. >
   f. > g. < h. > i. > j. <

3. a. -6 < -5 < 0 < 4
   b. -5 < -1 < 1 < 2
   c. -100 < -8 < -7 < -4

4. a. 3°C b. 9°C c. 2°C d. 15°C
   e. 26°C f. 22°C g. 12°C h. 3°C
   i. 14°C j. 12°C

5.

before	12°C	4°C	-3°C	-9°C	-11° C	-5°C
change	drops 15°C	drops 8°C	drops 10°C	rises 10°C	rises 6°C	rises 7°C
now	-3°C	-4°C	-13°C	1°C	-5°C	2°C

6. a. False. Zero is neither.
   b. False. You get zero.
   c. True.
   d. False. It is not negative, neither positive.
   e. True.
   f. True. Zero is its own opposite.

Integers, p. 6

1. a. ⁻$250 b. ⁻120 ft c. $115
   d. ⁻12 yd e. ⁻9° F f. 185 m

2. a. > b. > c. > d. < e. < f. > g. > h. > i. > j. <

3. a. ⁻1, 0, 1, 3, 4, 7, 10 b. ⁻1, 0, 1 c. ⁻8, ⁻5
   d. 0, 1, 3, 4, 7, 10 e. ⁻3, ⁻1, 0, 1, 3
   f. 4, 3, 1, 0, ⁻1, ⁻3, ⁻4, ⁻5, ⁻8

4. a. 4 b. ⁻2 c. ⁻7 d. ⁻4 e. ⁻6 f. 5
   g. ⁻40 h. 10 i. ⁻900

5. a. ⁻65, ⁻60, ⁻55, ⁻50, ⁻45, ⁻40
   b. ⁻14, ⁻11, ⁻8, ⁻5, ⁻2, 1
   c. ⁻3, ⁻6, ⁻9, ⁻12, ⁻15, ⁻18

6. a. ⁻12, 0, 12, 21 b. ⁻5, ⁻4, 2, 3 c. ⁻13, ⁻7, ⁻1, 3
   d. ⁻15, ⁻9, ⁻5, 5 e. ⁻10, 0, 6, 10 f. ⁻9, ⁻4, ⁻3, 3

Adding Integers, p. 7

1. a. 12; ⁻12 b. ⁻8; ⁻30 c. ⁻44; 28 d. ⁻520; ⁻60

2. a. ⁻7 b. ⁻52 c. ⁻16

3. a. ⁻2 + 2 = 0. b. 3 + ⁻3 = 0.
   c. ⁻4 + 3 = ⁻1. d. 4 + ⁻2 = 2.
   e. 3 + ⁻5 = ⁻2. f. 1 + ⁻5 = ⁻4.
   g. ⁻3 + 4 = 1. h. ⁻6 + 4 = ⁻2.

4. a. 1; 3; 5 b. 0; 0; ⁻10 c. 9; ⁻4; ⁻6 d. 3; 2; 1

5.

a.	b.	c.	d.
5 + ⁻6 = ⁻1	2 + ⁻2 = 0	6 + ⁻9 = ⁻3	⁻4 + 6 = 2
⁻9 + 8 = ⁻1	⁻10 + 10 = 0	⁻4 + 1 = ⁻3	9 + ⁻7 = 2
3 + ⁻4 = ⁻1	99 + ⁻99 = 0	1 + ⁻4 = ⁻3	⁻1 + 3 = 2

6. a. ⁻7 b. 12 c. 6

Add and Subtract Integers, p. 8

1.	Addition sentence:
a. Now it is 1°C.	⁻3 + 4 = 1
b. Now it is ⁻3°C.	⁻5 + 2 = ⁻3
c. Now it is ⁻5°C.	⁻9 + 4 = ⁻5
d. Now it is 6°C.	⁻1 + 7 = 6
e. Now it is 3°C.	⁻2 + 5 = 3
f. Now it is ⁻7°C.	⁻10 + 3 = ⁻7

2.	Subtraction sentence:
a. Now it is ⁻1°C.	3 − 4 = ⁻1
b. Now it is ⁻3°C.	7 − 10 = ⁻3
c. Now it is ⁻3°C.	5 − 8 = ⁻3
d. Now it is ⁻6°C.	⁻1 − 5 = ⁻6
e. Now it is ⁻14°C.	⁻7 − 7 = ⁻14
f. Now it is ⁻12°C.	⁻10 − 2 = ⁻12

3.

a.	b.
5 − 4 = 1 5 − 5 = 0 5 − 6 = ⁻1 5 − 7 = ⁻2 5 − 8 = ⁻3	⁻4 − 0 = ⁻4 ⁻4 − 1 = ⁻5 ⁻4 − 2 = ⁻6 ⁻4 − 3 = ⁻7 ⁻4 − 4 = ⁻8
c.	**d.**
⁻3 + 0 = ⁻3 ⁻3 + 1 = ⁻2 ⁻3 + 2 = ⁻1 ⁻3 + 3 = 0 ⁻3 + 4 = 1	⁻2 + 2 = 0 ⁻2 + 3 = 1 ⁻2 + 4 = 2 ⁻2 + 5 = 3 ⁻2 + 6 = 4'

4. a. Now he is at **-5** ft. ⁻20 + 15 = ⁻5.
   b. Now he **owes $5**. 15 − 20 = ⁻5.
   c. Now he **has $5**. ⁻15 + 20 = 5.
   d. Now the ball is at **-5 ft**. 15 − 20 = -5.
   e. Now the temperature is **5°C**. 20 − 15 = 5.

Number Line Jumps, p. 9

1. a. You end up at 4. -2 + 6 = 4.
   b. You end up at 1. -4 + 5 = 1.
   c. You end up at -5. -7 + 2 = -5.
   d. You end up at -6. -10 + 4 = -6.

2. a. You end up at -4. 2 − 6 = -4.
   b. You end up at -5. 5 − 10 = -5.
   c. You end up at -8. -4 − 4 = -8.
   d. You end up at -10. -7 − 3 = -10.

3. a. -2; -10; -4; -20. b. -10; -14; -20; -52
   c. 3; -5; -50; -6 d. 0; 30; -6; -1

6. a. 8 b. 8 c. 4 d. 5 e. 10 f. 11 g. 8 h. 14

Opposites and Absolute Value, p. 10

1.

Integer	−5	8	6	−11	−19	−4	4	0
Opposite	5	−8	−6	11	19	4	−4	0
Absolute value	5	8	6	11	19	4		0

2. a. −7 b. |−7| = 7 c. −|7| = −7 d. −|−7| = −7

3. a. −12 b. 6 c. 1 d. −3 e. 1 f. −1

4. a. 10 and −10 b. 55 and −55
c. 3 and −3 d. 5 and −5

5. a. 9 b. −9 c. −9 d. −9 e. 12 f. 20
g. −4 h. 4 i. −11 j. 11 k. 6 l. 6

6. a. < ; difference 9 b. > ; difference 7
c. < ; difference 2
d. >; difference 7 e. > difference 11

7.

	a.	b.	c.	d.	e.
	6 > 3	5 > 2	8 > 6	10 > 3	4 < 7
Difference:	3	3	2	7	3

Add Integers 1, p. 11

1. a. 5 b. ¯6 c. ¯1 d. 1 e. ¯2 f. ¯6 g. 1 h. ¯7

2. a. 1 b. ¯14 c. 3

3. a. 6 + ¯8 = ¯2 b. ¯4 + ¯5 = ¯9 c. ¯4 + 8 = 4
d. 7 + ¯8 + ¯1 = ¯2 e. 3 + ¯5 + 1 = ¯1
f. ¯10 + ¯5 + 3 + 6 = ¯6

4. a. 1 b. ¯11 c. ¯1

5. $20 + ¯$12 + ¯$15 + ¯$12 + $25 + ¯$10 + $50 = $46.
No, Mark did not owe money.

6. a.	b.	c.	d.
6 + (-3) = 3	-10 + 6 = -4	(-3) + (-4) = -7	(-90) + 10 = -80
6 + (-4) = 2	-10 + 7 = -3	(-3) + (-3) = -6	(-90) + 15 = -75
6 + (-5) = 1	-10 + 8 = -2	(-3) + (-2) = -5	(-90) + 20 = -70
6 + (-6) = 0	-10 + 9 = -1	(-3) + (-1) = -4	(-90) + 25 = -65
6 + (-7) = -1	-10 + 10 = 0	(-3) + 0 = -3	(-90) + 30 = -60
6 + (-8) = -2	-10 + 11 = 1	(-3) + 1 = -2	(-90) + 35 = -55
6 + (-9) = -3	-10 + 12 = 2	(-3) + 2 = -1	(-90) + 40 = -50

Add Integers 2, p. 12

Add two integers with same sign:

Find the sum of their absolute values.
The answer has the same sign as the addends.

Add two integers with different sign:

Find the difference of their absolute values.
The answer has the same sign as the integer with larger absolute value.

1. a. ¯32 b. 250 c. ¯250

2. a. 10, 0, 5 b. ¯1, 4, ¯8 c. ¯6, ¯40, ¯11

3. a. 15 b. ¯387 c. ¯269

4. a. x = ¯9 b. y = 5 c. z = 10
d. x = ¯7 e. y = ¯1 f. z = ¯572

5. a. ¯5, 9, ¯145
b. Answers will vary. For example:
¯6, ¯8 + 4 + 4 = 0, 450 + ¯225 + ¯225 = 0
c. ¯2, ¯10, 151
d. Answers will vary. For example:
0, ¯7 + 6 + 2 = 1, 6 + ¯4 + ¯1

6. a. 646 b. ¯673 c. ¯87

Explore Subtraction, p. 13

1.

a.

x	5 − x
2	5 − 2 = 3
3	5 − 3 = 2
4	5 − 4 = 1
5	5 − 5 = 0
6	5 − 6 = ⁻1
7	5 − 7 = ⁻2
8	5 − 8 = ⁻3
9	5 − 9 = ⁻4

b.

y	100 − y
60	100 − 60 = 40
70	100 − 70 = 30
80	100 − 80 = 20
90	100 − 90 = 10
100	100 − 100 = 0
110	100 − 110 = ⁻10
120	100 − 120 = ⁻20
130	100 − 130 = ⁻30

c.

x	500 − x
300	500 − 300 = 200
350	500 − 350 = 150
400	500 − 400 = 100
450	500 − 450 = 50
500	500 − 500 = 0
550	500 − 550 = ⁻50
600	500 − 600 = ⁻100
650	500 − 650 = ⁻150

2.

a.

x	6 − x
3	6 − 3 = 3
2	6 − 2 = 4
1	6 − 1 = 5
0	6 − 0 = 6
-1	6 − (⁻1) = 7
-2	6 − (⁻2) = 8
-3	6 − (⁻3) = 9
-4	6 − (⁻4) = 10

b.

x	10 − x
4	10 − 4 = 6
6	10 − 6 = 4
8	10 − 8 = 2
10	10 − 10 = 0
12	10 − 12 = ⁻2
14	10 − 14 = ⁻4
16	10 − 16 = ⁻6
18	10 − 18 = ⁻8

c.

x	30 − x
10	30 − 10 = 20
20	30 − 20 = 10
30	30 − 30 = 0
40	30 − 40 = ⁻10
50	30 − 50 = ⁻20
60	30 − 60 = ⁻30
70	30 − 70 = ⁻40
80	30 − 80 = ⁻50

3. a. 2 − 6 = ⁻4 b. ⁻1 − 4 = ⁻5 c. 1 − 10 = ⁻9 d. ⁻17 − 7 = ⁻24

4.. a. ⁻5, ⁻3, ⁻30, ⁻332 b. 5, 21, 55, 785 c. 2, 3, 0, ⁻105

Subtract Integers, p. 14

1. You can ALWAYS change any subtraction into addition: Instead of subtracting a number, add its opposite.

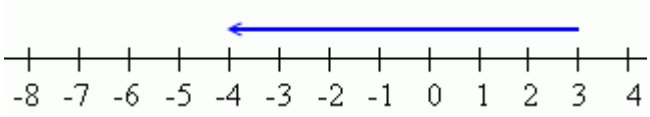
1. a. 3
   b. (−4) − 11 = −4 + (−11) = ⁻15
   c. 7 − (−3) = 7 + 3 = 10
   d. (−6) − (−10) = (−6) + 10 = 4

2. a. ⁻10, ⁻20, ⁻72 b. 10, 20, 72
   c. 4, 6, 30

3. a. 4 − ⁻2 = 6 b. ⁻9 − 11 = ⁻20
   c. ⁻5 − ⁻20 = 15 d. ⁻8 − ⁻100 = 92

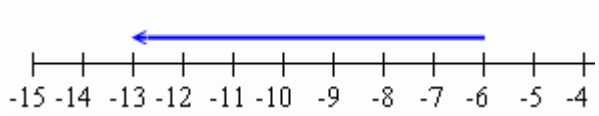
4. a. (⁻6) − 4 = ⁻10 b. (⁻6) + 4 = ⁻2 c. 6 + 4 = 10

5. a.



3 − 7 = −4

b.



−6 − 7 = −13

6. Temperature falls, so we subtract. T refers to *temperature*.

Initial T	2°C	3°C	-1°C	3°C	-2°C	5°C	-8°C
Drop	3°C	4°C	2°C	4°C	2°C	8°C	2°C
Final T	-1°C	-1°C	-3°C	-1°C	-4°C	-3°C	-10°C
Subtraction sentence	2 − 3 = −1	3 − 4 = ⁻1	⁻1 − 2 = ⁻3	3 − 4 = ⁻1	⁻2 − 2 = ⁻4	5 − 8 = ⁻3	⁻8 − 2 = ⁻10

7. a. ⁻8, ⁻7, ⁻120 b. ⁻15, ⁻10, ⁻180 c. ⁻13, 24, 36

Add/Subtract Roundup, p. 15

1. These 'rules' below are just examples; the student might have different wordings of these same ideas.

a. Adding two negative numbers: Add their absolute values, and set the answer as negative.

b. Adding a negative and a positive number: subtract their absolute values, and check which had the greater absolute value, and set that sign to the answer.

c. Subtracting a positive number:
Move on the number line towards the left.

d. Subtracting a negative number:
This is the same as adding the positive number.

.2. a. ⁻4, 18, ⁻30 b. 110, ⁻20, 26
c. ⁻20, 4, 7

3. a. ⁻21 b. 97 c. 111

4. a. ⁻9 b. ⁻3 c. ⁻24

5. a. x = ⁻1, y = ⁻4 b. a = ⁻1, b= ⁻5
c. z = 4, w = 4

6. The quickest way is to add all negative numbers together first, and also all positive ones. Then sum up those.
a. 31 + (⁻21) = 10 b. 45 + (⁻120) = ⁻75

7. a. ⁻3 b. ⁻2

8. a. ⁻9, 8 b. ⁻6, 4 c. ⁻13, ⁻8 d. ⁻4, ⁻4

Multiply Integers, p. 16

Multiplication by a whole number is repeated addition:

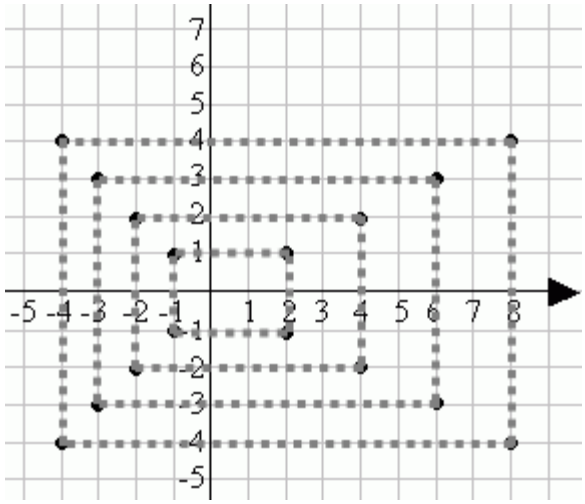
4 × ⁻3 = ⁻3 + ⁻3 + ⁻3 + ⁻3 = ⁻12

1. a. -21; -10; -40 b. -24; -100; -100
c. 0; -90; -400 d. 0; -280; -80

2. a. -10 b. -25 c. -12 d. -110

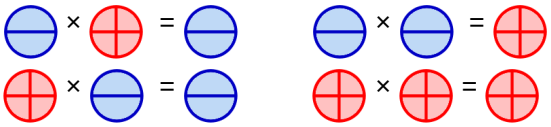
3. e. The side lengths of the three new rectangles also are two, three, and four times the side lengths of the original rectangle.
f. Areas are: 6 square units, 24 square units, 54 square units, 96 square units.
g. 1:4
h. 1:9
i. 1:16

3. a.-d.



Multiply Integers, p. 17

1.



2. a. $4 \times {}^{-}2 = {}^{-}8$ b. $2 \times {}^{-}11 = {}^{-}22$ c. $3 \times {}^{-}100 = {}^{-}300$

3. a. ⁻8 b. ⁻22 c. 30 d. 100
e. ⁻16 f. 14 g. ⁻30 h. 1

4. See below.

5. a. 40 b. ⁻95 c. 75

6. a. ⁻20 b. 20 c. 120 d. ⁻40

7. a. 100 b. 0 c. 300 d. 60 e. ⁻1,500 f. ⁻120

8. a. $y = {}^{-}10$ b. $a = {}^{-}9$ c. $z = 7$
d. $w = 16$ e. $x = 11$ f. $z = {}^{-}8$

4. a.

x	2x
3	2(3) = 6
2	2(2) = 4
1	2(1) = 2
0	2(0) = 0
−1	2(−1) = −2
−2	2(−2) = −4
−3	2(−3) = −6
−4	2(−4) = −8

b.

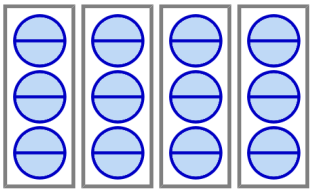
y	(−3)y
3	(−3)(3) = −9
2	(−3)(2) = −6
1	(−3)(1) = −3
0	(−3)(0) = 0
−1	(−3)(−1) = 3
−2	(−3)(−2) = 6
−3	(−3)(−3) = 9
−4	(−3)(−4) = 12

c.

a	5a + 1
2	5(2) + 1 = 11
1	5(1) + 1 = 6
0	5(0) + 1 = 1
−1	5(−1) + 1 = −4
−2	5(−2) + 1 = −9
−3	5(−3) + 1 = −14
−4	5(−4) + 1 = −19
−5	5(−5) + 1 = −24

Divide Integers, p. 18

1. $-12 \div 4 = -3$.



2. a. 2 b. $21 \div (-7) = -3$; $-3 \times (-7) = 21$
c. $-20 \div (-5) = 4$; $4 \times (-5) = -20$
d. $-24 \div 6 = (-4)$; $(-4) \times 6 = -24$
e. $(-80) \div (-10) = 8$; $8 \times (-10) = (-80)$
f. $35 \div (-5) = (-7)$; $(-7) \times (-5) = 35$

3. a. ⁻2 b. ⁻10 c. 3 d. 100 e. ⁻40 f. 1 g. 2 h. ⁻1

4. a. ⁻22 b. ⁻18 c. 40 d. 10

5. a. ⁻1, ⁻1, 1, 1 b. ⁻7, ⁻7, 7, 7 c. ⁻94, ⁻94, 94, 94

6. a. 4 b. ⁻20 c. 10

7. a. 1 b. ⁻26 c. ⁻30

8. a. y= ⁻1000 b. $a = {}^{-}9$ c. $z = {}^{-}4$
d. $w = {}^{-}10$ e. $x = 7$ f. $m = {}^{-}48$

Integer Problems, p. 19

1. a. $^{-}753 + 37 = {}^{-}716$. His reign ended 716 BC.
b. $-4°C + 7°C = 3°C$
c. $-6°C - 6°C = -12°C$
d. $1°C - 5°C = -4°C$
e. $\$15 - \$50 = -\$35$
f. $(-\$45) + (-\$24) + (-\$20) + (-\$50) + (-\$10) = -\$149$
g. -80 ft $+ 20$ ft $- 40$ ft $- 25$ ft $+ 60$ ft $= -65$ ft, or depth of 65 ft.

2. a. ⁻352 b. ⁻60 c. 3,690 d. ⁻320
e. 2 f. 1 g. 6 h. ⁻36 i. 11

3. The sum of them is 14°C, and average is 2 °C.

4. a. Not true; the opposite of 4 is ⁻4, and the opposite of ⁻4 is 4.
b. True.
c. Not true. For example, ⁻9 is less than 2 but its absolute value is 9, which is more than 2.
d. True.
e. True.

Powers with Negative Base, p. 20

1. a. -8 b. $(-10)^2 = (-10) \times (-10) = 100$
   c. $(-4)^2 = (-4) \times (-4) = 16$
   d. $(-10)^3 = (-10) \times (-10) \times (-10) = -1{,}000$

2. a. 10; 100; 1,000; 10,000; 100,000; 1,000,000, 10,000,000; 100,000,000
   b. ¯10; 100; ¯1,000; 10,000; ¯100,000; 1,000,000; ¯10,000,000; 100,000,000
   c. 2; 4; 8, 16, 32, 64, 128, 256, 512
   d. ¯2, 4; ¯8; 16; ¯32; 64; ¯128; 256; ¯512

3. a. ¯64 b. 1 c. 25 d. ¯125 e. ¯32
   f. 10,000 g. ¯1 h. 81

4. a. 9,960 b. 36 c. ¯1,728 d. 0 e. 162 f. 4

5. a. ¯62 b. 53 c. ¯174 d. 20

6. a. 2 b. 5 c. 3 d. any even number

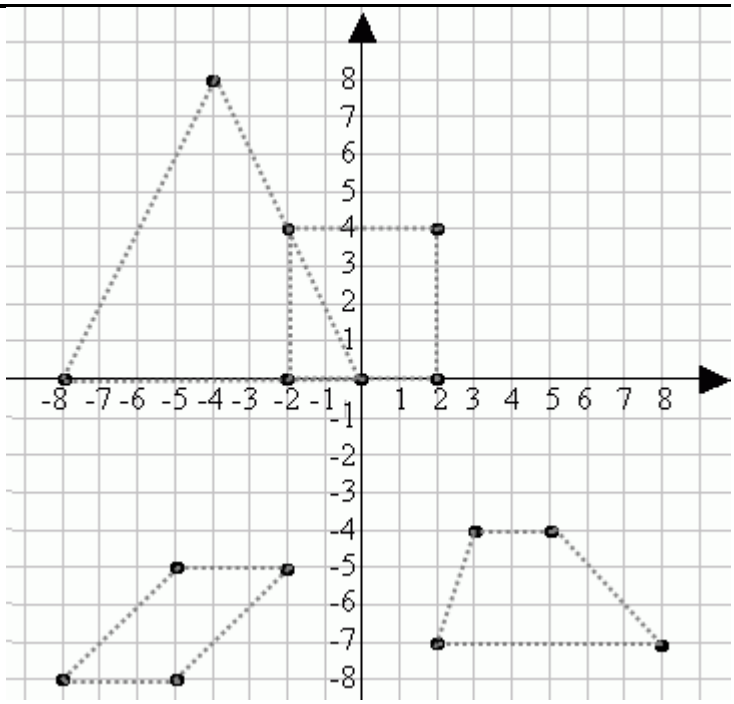
Coordinates, p. 21

1. A. (-7,0) B. (-4,1) C (1,3) D (6,2) E (-6, -3)
   F (-3, -2) G (-2, -5) H (0, -4) I (2, -2) J (5, -7)

2. a. Rectangle b. parallelogram
   c. trapezoid d. an isosceles triangle

3.

Point	Direction	New point
(-2,4)	two up, two left	(-4, 6)
(1,1)	3 down, 7 left	(-6, -2)
(0,-3)	four up, five left	(-5, 1)
(2,-5)	6 up, 6 left	(-4, 1)
(0,0)	2 down, 1 right	(1, -2)
(5,8)	14 down, 7 left	(-2, -6)
(-5,-6)	10 up, 10 right	(5, 4)
(-2,-2)	5 up, 7 right	(5, 3)



Movements in Coordinate Grid, p. 22

1. (-7, 4), (-2, 4), (-7, 7), (-2, 7)

2. (-7, -3), (-2, -3), (-7, 0), (-2, 0)

3. (3, 7), (8, 7), (3, 10), (8, 10)

4. New vertices: (0, -2), (2, -3), (1,2)

5. (-3, -3), (1, -3), (2, -1), (-2, -1)

6. (-1, 4). (110,230)

Functions in Coordinate Grid 1, p. 23

1. a. To get the y-coordinate, add 2.

x	y
-1	1
0	2
1	3
2	4
3	5
4	6

b. To get the y-coordinate, add 3.

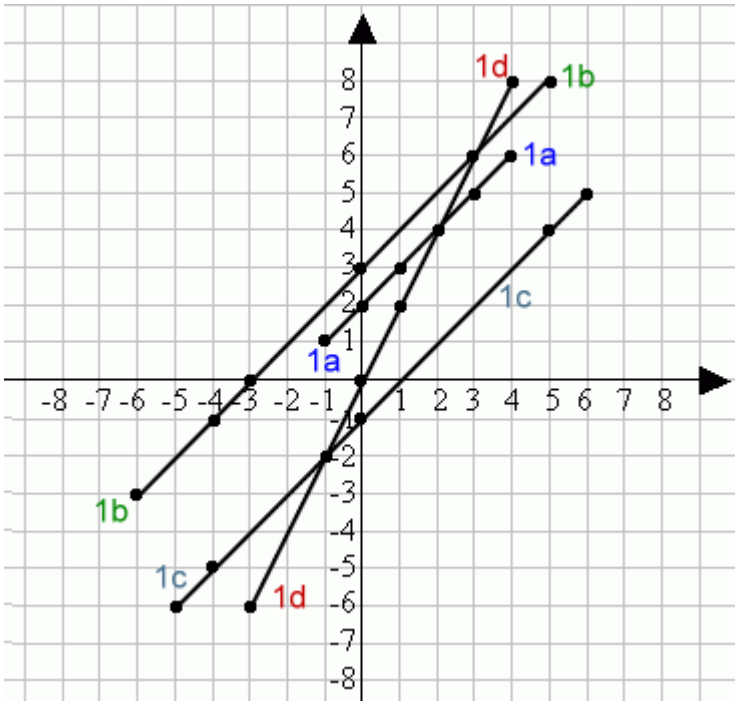
x	y
-6	-3
-4	-1
-3	0
0	3
3	6
5	8

c. To get the y-coordinate, subtract 1.

x	y
-5	-6
-4	-5
-1	-2
0	-1
5	4
6	5

d. To get the y-coordinate, multiply by 2.

x	y
-3	-6
-1	-2
0	0
1	2
2	4
4	8

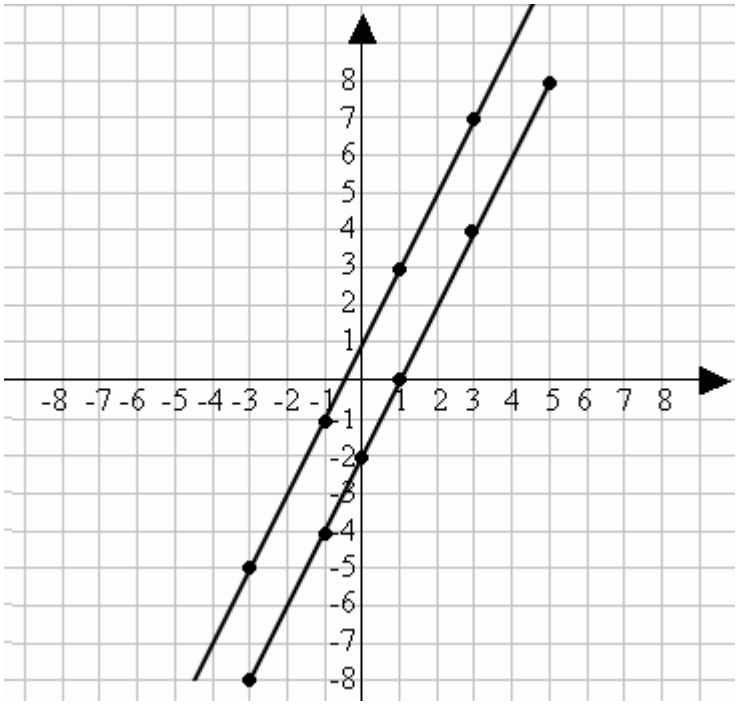


2. a.

x	y = 2x + 1
-5	-9
-3	-5
-1	-1
1	3
3	7
5	11

b.

x	y = 2 x – 2
-3	-8
-1	-4
0	-2
1	0
3	4
5	8



Functions in Coordinate Grid 2, p. 24

1. a. To get the y-coordinate, multiply by 2 and add 2.

x	y
-4	-6
-3	-4
-2	-2
-1	0
0	2
1	4
2	6
3	8
4	10

b. To get the y-coordinate, divide by 2.

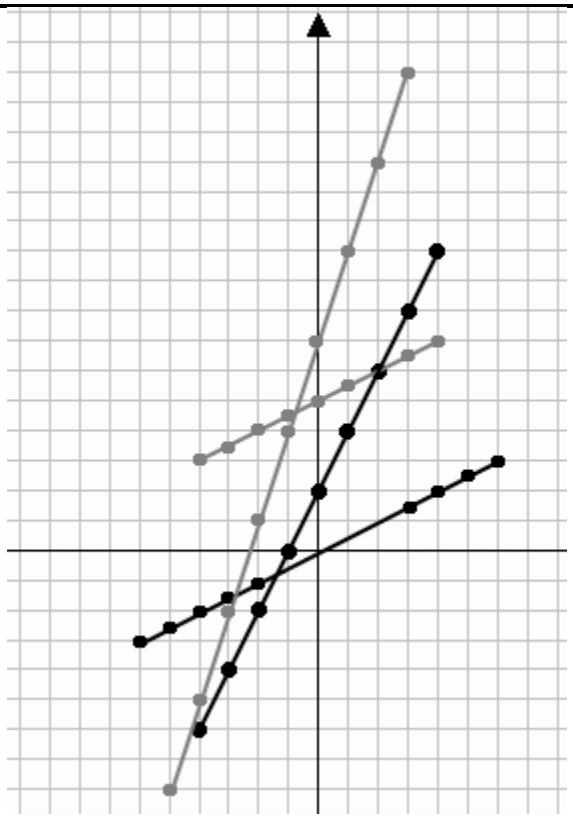
x	y
-6	-3
-5	-2.5
-4	-2
-3	-1.5
-2	-1
3	1.5
4	2
5	2.5
6	3

2. a.

x	y = 3x + 7
-5	-8
-4	-5
-3	-2
-2	1
-1	4
0	7
1	10
2	13
3	16

b.

x	y = 1/2x + 5
-4	3
-3	3 1/2
-2	4
-1	4 1/2
0	5
1	5 1/2
2	6
3	6 1/2
4	7



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