

Unit 1

Getting started with Algebra

In this unit, you embark on your journey through algebra. The unit begins forming the foundation for several important mathematical ideas, “learning to learn” ideas, and classroom routines and tools that you will use throughout the program:

- Classroom routines for collaboration, homework, and communication
- Mathematical patterns, variables, algebraic expressions, graphing; input-output relationships, mathematical generalizations
- The malleability of intelligence and the importance of effort in promoting brain development
- Problem solving and problem-solving strategies

OUTLINE

Topic 1: Exploring problem-solving strategies

This topic introduces you to tools and strategies for collaborating to solve problems.

In this topic, you will:

- Introduce yourself and learn about your teacher, your classmates, and this course
- Learn and practice norms and routines to help you collaborate and learn
- Explore, apply, and share problem-solving strategies, and reflect on your solutions
- Work with important algebra ideas, including variables, patterns, and solutions

Topic 2: Getting smarter through algebraic reasoning

This topic introduces you to the ideas of malleable intelligence and brain growth through learning. You will learn about the concept of working harder to get smarter, and then apply this idea to learning mathematics. You will continue to develop problem-solving strategies as you apply a problem-solving routine to investigate and answer a variety of problems.

In this topic, you will:

- Examine and reflect on your perceptions of your math ability
- Learn how intelligence is malleable, not fixed
- Understand how working on challenging problems affects the brain
- Learn and apply a routine to help with mathematical problem solving
- Develop conjectures with signed numbers using pattern recognition

Topic 3: Foundations of algebra

In this topic you continue to develop foundational algebraic ideas as you consider ideas of variables, expressions, graphing, and the distributive property.

In this topic, you will:

- Use variables to represent unknowns
- Create, interpret, and evaluate algebraic expressions
- Use the distributive property
- Use variables to generalize input-output relationships
- Plot points on a graph
- Make graphs from tables
- Interpret the meaning of points on a graph

EXPLORING PROBLEM-SOLVING STRATEGIES

Lesson 1.1 Collaborating around algebra

1.1 OPENER

In this activity and throughout today's lesson, you will be solving shape equation puzzles. These puzzles draw on the mathematics skills you learned in middle school, while at the same time developing thinking that is important for algebra.

1. "Crack the code" to solve this puzzle by figuring out the numbers represented by the square and the triangle.

$$\blacksquare - \blacktriangle = 4$$

$$\blacksquare \times \blacktriangle = 12$$

Answer: $\blacksquare = \underline{\hspace{2cm}}$; $\blacktriangle = \underline{\hspace{2cm}}$

How did you solve the puzzle?

2. Solve the next puzzle. [Note: The values of shapes can be different in this new problem.]

$$\blacksquare + \blacktriangle = 10$$

$$\bullet - \bullet + \blacksquare = 4$$

$$\bullet - \blacktriangle = 2$$

Answer: $\blacksquare = \underline{\hspace{2cm}}$; $\blacktriangle = \underline{\hspace{2cm}}$; $\bullet = \underline{\hspace{2cm}}$

How did you solve the puzzle?

1.1 CORE ACTIVITY

Work with your partner to figure out the number that each shape represents. Don't forget to practice good teamwork!

Shape Equation Puzzle 1		Solution and explanation
Clue 1	$\triangle + \diamond = \hexagon$	
Clue 2	$\triangle + \hexagon = 13$	
Clue 3	$\diamond - \triangle = 1$	
Clue 4	$2 + \diamond = 7$	

Shape Equation Puzzle 2		Solution and explanation
Clue 1	$\diamond + \diamond = \star$	
Clue 2	$\triangle - \diamond = \star$	
Clue 3	$\star \times 3 = 24$	
Clue 4	$\frac{\triangle}{3} = \diamond$	

Shape Equation Puzzle 3		Solution and explanation
Clue 1	$\bigcirc \times \nabla = \nabla$	
Clue 2	$\bigcirc + \bigcirc = \triangle$	
Clue 3	$\triangle + \nabla = \triangle$	
Clue 4	$\bigcirc \times \bigcirc = \triangle$	

Shape Equation Puzzle 4		Solution and explanation
Clue 1	$\triangle + \diamond + \diamond = \hexagon$	
Clue 2	$\triangle + 1 + \triangle = \diamond$	
Clue 3	$\triangle + \diamond + \hexagon = 11$	
Clue 4	$\hexagon - \diamond = 4 \times \triangle$	

1.1 CONSOLIDATION ACTIVITY

For the following problems, work with your partner to rewrite the shape equations using letters as variables. You can decide which letters to use for your variables.

Example:

Equations using shapes	Equations using letters
$\blacksquare - \blacktriangle = 4$ $\blacksquare \times \blacktriangle = 12$	

Puzzle 1	
Equations using shapes	Equations using letters
$\triangle + \diamond = \hexagon$	
$\triangle + \hexagon = 13$	
$\diamond - \triangle = 1$	
$2 + \diamond = 7$	

Puzzle 2	
Equations using shapes	Equations using letters
$\diamond + \diamond = \star$	
$\triangle - \diamond = \star$	
$\star \times 3 = 24$	
$\frac{\triangle}{3} = \diamond$	

HOMEWORK 1.1

Notes or additional instructions based on whole-class discussion of homework assignment:

1. Solve the shape equation puzzles.

Shape Equation Puzzle 1		Solution and explanation
Clue 1	$\triangle + \diamond + \star = 22$	
Clue 2	$\triangle \div 2 = 4$	
Clue 3	$\star - \diamond = \triangle$	
Clue 4	$2 + \diamond + \diamond = \triangle$	





Shape Equation Puzzle 2		Solution and explanation
Clue 1	$\star = \diamond$	
Clue 2	$(\star + \diamond) \times \triangle = 20$	
Clue 3	$\triangle \times \triangle \times \diamond = 2 \times \star \times \triangle$	
Clue 4	$\star \times \diamond = 25$	

2. Create your own shape equation puzzle. In the space below the table, provide the solution and show that your solution works.

My Shape Equation Puzzle	
Clue 1	
Clue 2	
Clue 3	
Clue 4	


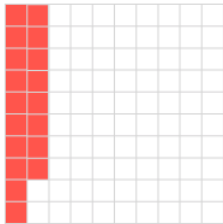
Solution with explanation showing that the solution works:

3. Rewrite the shape equations using letters as variables. You can decide which letters to use for your variables.

Shape Equation Puzzle 3	
Equations using shapes	Equations using letters
	
	
	
	

4. As a class, you discussed a routine for what you will do each day when you come into the classroom. In the space below, state the routine.

STAYING SHARP 1.1

PRACTICING ALGEBRA SKILLS & CONCEPTS	<p>1. Solve the shape equation puzzle:</p> $\bullet + \blacksquare = 14$ $\bullet - \blacksquare = 2$ $\bullet = \underline{\hspace{2cm}}$ $\blacksquare = \underline{\hspace{2cm}}$	<p>2. Continue this pattern of shapes for three more steps.</p> <div style="text-align: center;">  </div> <p>Explain the pattern or how you continued it:</p>
PREPARING FOR UPCOMING LESSONS	<p>3. If the large square represents one whole, what fraction is represented by the shaded area?</p> <div style="text-align: center;">  </div> <p>Answer with supporting work:</p>	<p>4. To get ready for a field trip, students and adults were put into groups. For every 8 students in a group, there were 3 adults. In total, in all the groups, there were 48 students. How many adults were also on the field trip?</p>
REVIEWING PRE-ALGEBRA IDEAS	<p>5. Estimate, to the nearest dollar, the total cost:</p> $\$23.45 + \$3.98 + \$16.66 + \12.08 <p>Answer with supporting work:</p>	<p>6. A rectangle has length 7 inches and width 3 inches. Sketch the rectangle and determine its area, with units.</p> <p>Answer with supporting work:</p>

Lesson 1.2 Collaboration and problem solving

1.2 OPENER

Think back to some specific group learning experiences with problem solving. For example, think about your work in Lesson 1 in solving the shape equation puzzles. You can also think about experiences in middle school or junior high school.

1. Think of a time when you successfully worked with a partner or in a group on an activity. List five or more things that made your collaboration successful.
2. Think of a time when you struggled to work with a partner or in a group on an activity. List five or more things that made your collaboration difficult.



1.2 CORE ACTIVITY

1. As a class, you will establish some criteria for quality partner work. Summarize the ideas from the class discussion in the space below.

Criteria for Quality Partner Work

2. McKenna and Lara work for their uncle Eddie, who repairs skateboards and bicycles. Uncle Eddie is leaving for a vacation to the Amazon. He asks the girls to order 54 new wheels for the 21 skateboards and bicycles in his repair shop. How many bicycles and how many skateboards are in Uncle Eddie's shop?

From the list of facts, circle those you need to solve Lara and McKenna's problem.

- McKenna and Lara's uncle is named Eddie.
- There are 21 skateboards and bicycles in the shop.
- Uncle Eddie went to the Amazon for 6 weeks.
- A bicycle has 2 wheels and a skateboard has 4 wheels.
- 54 new wheels are needed.



3. How many bicycles and how many skateboards are in the shop? Show your work.

4. Explain how you and partner found the answer. [If you did not find an answer, explain what you did to try to find an answer.]

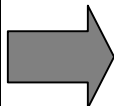
1.2 PROCESS HOMEWORK

The Homework-Processing Routine

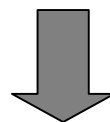
1**With Your Partner**




Review the homework and discuss:

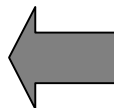
- ⇒ Compare your answers.
 - ◆ Did you both get the same answers?
 - ◆ If not, think things through—which solution is correct?
- ⇒ Compare how you solved the problem.
 - ◆ Did you both use the same approach?
 - ◆ If you used different approaches, look at the other person's approach. How is it similar to yours? How is it different?
- ⇒ Discuss how you would explain this problem if you are asked to do so.

**2****As a Class**

- ⇒ Your teacher will identify several problems to review as class.
 - ◆ Be prepared to present a solution with your partner
 - ◆ Think about questions you have related to the problems
- ⇒ Participate in the whole-class discussion of the featured problems.
- ⇒ Correct additional problems if your teacher posts solutions.

**3****By Yourself**

- ⇒ Correct problems that you got wrong.
 - ◆ Make sure you understand why you got a problem wrong.
- ⇒ USE A RED PEN TO MAKE YOUR CORRECTIONS.
- ⇒ Mark a “stoplight indicator” at the top of the homework assignment:
 -  = I understand all of the ideas in the homework.
 -  = I understand some/most of the ideas.
 -  = I don't understand most of the ideas.

**4****Finish**

- ⇒ Place your corrected homework assignment in your two-pocket folder. Be sure to place it on the IN side.
- ⇒ Remove any assignment on the OUT side of the folder and place it in your notebook. Your teacher has looked at these assignments to monitor your understanding and has checked them in.

1.2 CONSOLIDATION ACTIVITY

1. Problem solving for these friends was similar to exploring a maze.

a. Why is a maze a good model for problem solving?



b. Describe another model for problem solving.

2. Mike felt confident he could make it through the maze, while his friends weren't so sure. How confident are you with problem solving in math? Explain your answer.

3. Describe some things that you can do if you are stuck when solving a problem.

4. Did you and your partner work well together solving the Bike and Skateboard Problem? Which things on the "Criteria for Quality Partner Work" chart did you do well? Which things can you and your partner work to improve?

5. In addition to learning algebra in this course, you will also focus on "learning to learn" topics. Why might this be helpful?

HOMEWORK 1.2

Notes or additional instructions based on whole-class discussion of homework assignment:

1. Solve the shape equation puzzles.

Shape Equation Puzzle 1		Solution and explanation
Clue 1	$2\triangle - 1 = 5$	
Clue 2	$\triangle + \square = 1$	
Clue 3	$\triangle - 2\square = \star$	

Shape Equation Puzzle 2		Solution and explanation
Clue 1	$\diamond + 2 = 5\bigcirc$	
Clue 2	$3\bigcirc = 12$	
Clue 3	$2\bigcirc = \diamond + 2\triangle$	

2. For each of the following classroom routines, list some important norms, behaviors, or expectations. You do not need to list all of the things that you discussed in class in connection with each category.

Routine	Some important norms, behaviors, and expectations
a. Working in groups	
b. Reviewing homework with my partner	

STAYING SHARP 1.2

PRACTICING ALGEBRA SKILLS & CONCEPTS	<div>1. Solve the shape equation puzzle:</div> <div><div><div>■</div><div>+</div><div>★</div><div>=</div><div>8</div></div><div><div>■</div><div>•</div><div>★</div><div>=</div><div>15</div></div><div><div>■</div><div>=</div><div>_____</div></div><div><div>★</div><div>=</div><div>_____</div></div></div>	<div>2. A pattern of letter arrangements begins with the 3 steps below. First, continue the pattern for 3 more steps:</div> <div><div>1. A</div><div>2. ABA</div><div>3. ABCBA</div><div>4. _____</div><div>5. _____</div><div>6. _____</div></div> <div>How many letters are in each step above?</div> <div>How many letters do you think there will be in step 10?</div>
PREPARING FOR UPCOMING LESSONS	<div>3. If it rained on 3 days out of 20 days, on what percent of the days did it rain?</div>	<div>4. Gina wants to ride the waves at Flagship Pier in Galveston, so she rents a surfboard for \$5 per hour plus an initial, or beginning, payment of \$20. For how many hours can Gina rent a surfboard if she has \$90?</div>
REVIEWING PRE-ALGEBRA IDEAS	<div>5. Meghan works at an ice cream shop and makes \$8.25 per hour. Lucas works for a landscaper and makes \$6.50 per hour. How much more does Meghan make each hour?</div>	<div>6. A rectangle has a perimeter of 26 cm, and one of its sides has a length of 5 cm. Sketch the rectangle and label all of its side lengths.</div>

Lesson 1.3 Mathematical presentations

1.3 OPENER

Today, you and your partner will present your solution to the Bike and Skateboard Problem.

Think about some things that make a good mathematical presentation.

- List some things that you think are important for good **written** presentations.
- List some things that you think are important for good **oral** presentations.

Characteristics of Good Written Presentations	Characteristics of Good Oral Presentations

1.3 CORE ACTIVITY

1. Discuss the following questions with your partner. Be prepared to share the ideas that you discuss with the class.
 - What does it mean to be a good audience member?
 - What are some characteristics of a good audience member?
2. The quality of your presentations will be judged using the “Class Presentation Criteria.” This list includes the criteria for high-quality class presentations.

Class Presentation Criteria
<ul style="list-style-type: none">• Include a clear write-up of your solution to the problem• Include a clear, concise explanation as to why you believe your answer is correct• Include a clear, concise explanation of solution strategy• Both partners participate in presentation• Both partners use strong, clear voices when making the presentation; both partners employ good posture and make eye contact

3. Work with your partner to prepare your presentation. Also, think about what each of you will say when you present your solution to the class.

Your oral presentation should include:

- ◆ Your solution
- ◆ How you know your solution is correct
- ◆ What you did to get your solution

Your presentation should be brief. You do not have to go through every step in your thought process. Summarize your approach and describe some of your thinking.

Uncle Eddie asked McKenna and Lara to order 54 new wheels for the 21 skateboards and bicycles in his repair shop. How many bicycles and how many skateboards are in Uncle Eddie's shop?	
There are <input type="text"/>	bicycles in the shop.
There are <input type="text"/>	skateboards in the shop.

1.3 CONSOLIDATION ACTIVITY

You will give a presentation with your partner about the Bike and Skateboard Problem. You will also listen to your other classmates give their presentations. Afterward, talk with your class about the presentations.

HOMEWORK 1.3

Notes or additional instructions based on whole-class discussion of homework assignment:

1. For the school play, adult tickets cost \$4 and children tickets cost \$2. Natalie is working at the ticket counter and just sold \$20 worth of tickets. What are all of the possible ticket combinations for \$20 worth of tickets?



2. Think about the presentation that you and your partner made in class today.
 - a. Write about one thing that you liked about the presentation — something that you feel you and your partner did well.
 - b. Write about one thing that you would like to improve the next time you make a presentation in math class.

3. Write a two-paragraph mini autobiography to help your teacher get to know you as a person, a learner, and a math learner.


In the space below, write the first paragraph. This paragraph will allow your teacher to get to know you as a person.

- By what name do you like to be called?
- Who do you live with? Do you have brothers or sisters?
- What are your interests, hobbies, and talents?
- When is your birthday?
- Anything else that you would like to share.

In the space below, write the second paragraph. This paragraph will allow your teacher to get to know you as a learner and as a math learner.

- What are your favorite subjects in school?
- What have been some of your experiences in math?
- Are there some topics in math that you like and are good at? Are there topics that you don't like or that have been difficult for you?
- Do you like working with others in school? Why or why not?
- Have you used a graphing calculator before?
- What are your thoughts about taking algebra this year?
- Anything else that you would like to share.

STAYING SHARP 1.3

PRACTICING ALGEBRA SKILLS & CONCEPTS	<p>1. If each piece of fruit stands for a number, it's possible to figure out the number for one of the pieces of fruit. Which fruit must equal a particular number, and why?</p>  <p>Answer with supporting work:</p>	<p>2. Compute these products with your calculator.</p> $1 \cdot 1 = \underline{\hspace{2cm}}$ $11 \cdot 11 = \underline{\hspace{2cm}}$ $111 \cdot 111 = \underline{\hspace{2cm}}$ $1111 \cdot 1111 = \underline{\hspace{2cm}}$ $11111 \cdot 11111 = \underline{\hspace{2cm}}$ $111111 \cdot 111111 = \underline{\hspace{2cm}}$ <p>Describe a pattern. (Do you think it will continue?)</p>
PREPARING FOR UPCOMING LESSONS	<p>3. A bag holds 1 blue block and 3 yellow blocks. How many green blocks should be added to the bag to make the probability of randomly selecting a green block from the bag equal to $\frac{1}{2}$?</p> <p>Answer with supporting work:</p>	<p>4. Joan likes to watch the birds in her yard. One morning, she notices that there are twice as many robins as bluejays. If she counted 21 birds in all, how many bluejays did she count?</p> <p>Answer with supporting work:</p>
REVIEWING PRE-ALGEBRA IDEAS	<p>5. Vipul is buying trail mix for an upcoming hike. The trail mix costs \$3.95 per pound and he is buying 2.01 pounds. Estimate the total cost of his trail mix.</p> <p>Answer with supporting work:</p>	<p>6. A cube is a three-dimensional solid with six square faces. Find the total surface area of a cube whose edges are each 5 feet in length.</p> <p>Answer with supporting work:</p>

Lesson 1.4 Building a mathematical toolbox

1.4 OPENER

Alejandra works at Cory's Coffee Shop after school on some days. A customer's order comes to \$4.70. The customer hands Alejandra a \$5 bill, so she gives the customer 30¢ in change. Because Alejandra loves math, she begins to think about the different ways in which she could have given the customer 30¢ in change.

- Using only quarters, dimes, and nickels, try to find all of the different ways that Alejandra could have given 30¢ in change.
- How could you organize your list in a way that makes it easy to keep track of the combinations?



1.4 CORE ACTIVITY

- Think about Lara's solution strategy:
 - Describe Lara's solution strategy.
 - Is the way that Lara solved the problem similar to the way that you and your partner solved the problem or different? In what ways?

Wheels	Bikes	Wheels	Skateboards
○ ○		○ ○ ○ ○	
○ ○		○ ○ ○ ○	
○ ○		○ ○ ○ ○	
○ ○		○ ○ ○ ○	
○ ○		○ ○ ○ ○	
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- Now think about McKenna's solution strategy.
 - Describe McKenna's solution strategy.

Bikes	Boards	Total bikes and boards	Bike wheels	Board wheels	Total wheels
1	20	21	2	80	82
2	19	21	4	76	80
7	14	21	14	56	70
12	9	21	24	36	60
14	7	21	28	28	56
15	6	21	30	24	54

- Is the way that McKenna solved the problem similar to the way that you and your partner solved the problem or different? In what ways?

3. Whose solution method—Lara’s or McKenna’s—made more sense to you? Explain why.

4. Suppose we change the Bike and Skateboard Problem slightly. In this new problem, Lara and McKenna must order 54 wheels for skateboards and bicycles. They know that there is at least one bicycle and at least one skateboard in the shop. How many bicycles and how many skateboards are in the shop?
 - a. What is different about this new problem compared to the original problem?
 - b. Will the solution to this new problem be the same as the solution to the original problem?

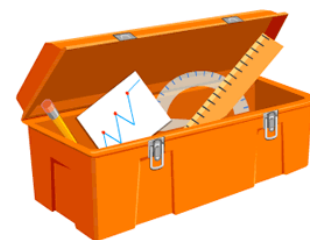
 - c. Work with your partner to solve this new problem. As you do, compare the strategies you use to the strategies you used to solve the original Bike and Skateboard problem. Show your work and solution in the space below.
 - d. When you solved this new Bike and Skateboard Problem, you may have discovered more than one correct solution. But are you sure that you found all of the possible solutions? Explain how you know that you found all possible solutions.

1.4 CONSOLIDATION ACTIVITY

You have seen a variety of strategies to solve the Bike and Skateboard Problem. Answer the following questions about problem-solving strategies.

1. In what ways does knowing more than one way to solve a problem help you?

2. How can knowing more than one solution strategy help you stay motivated to solve a problem if you are struggling and feel like giving up?



Think about the problems that you have solved in the course so far. Some examples include the Bike and Skateboard Problem and the shape equation puzzles.

3. With your partner, make a list of problem-solving strategies. In addition to thinking about strategies you used so far in this course, you can also think back to math problems that you worked on before high school, and the strategies that you used.

4. As a class, you will now create a poster of important problem-solving strategies. You and your partner will be asked to share some problem-solving strategies with the class. As your teacher records strategies on the poster, record the strategies in the space below.

Some Problem-Solving Strategies

5. Reflect on your learning so far in this course.
 - a. What are some of the main mathematical concepts that you have studied so far in the course?
 - b. List one or two concepts that you feel you understand well.

 - c. List one or two concepts that you feel you don't understand as well.
 - d. What are some things that you can do to improve your understanding of particular skills or ideas?

HOMEWORK 1.4

Notes or additional instructions based on whole-class discussion of homework assignment:

1. Timeless Toys builds tricycles and wagons. The owner noticed that there are several different ways that a customer could place an order for 5 items. For example, the customer could order all tricycles, or all wagons, or some combination of tricycles and wagons.

- a. For the case of a 5-item order, how many different ways are there to place an order? For each case, list the number of wheels that would be needed.



- b. If a customer ordered 5 items and the order had a total of 17 wheels, how many wagons were ordered? Show your work and explain your answer.

2. When solving a problem like the one above, what are some steps that you take? Think through your thought process. Then, list the steps that you take when you work to solve a problem.

3. Nathan has two types of stamps:

- 4¢ stamps (flower image)
- 8¢ stamps (mountain image)

We will assume that Nathan has sheets and sheets of these stamps and that he won't run out of them. Answer the following questions to consider how different combinations of stamps can be used.



- a. Nathan plans to mail a postcard that requires 40¢ in postage. List all of the different ways that he could use the two types of stamps to total 40¢ in postage.

- b. How can you be sure that you found all of the possible combinations?

- c. If Nathan used a total of 8 stamps for the 40¢ in postage, how many of each type of stamp did he use? Show your work and explain how you arrived at your answer.

4. Make up a problem similar to question 1. A classmate will try to solve your problem in class. You should still use the 4¢ stamps and the 8¢ stamps, but you can change the total postage that was on the envelope and the total number of stamps that were used. Be sure that your problem works out mathematically. [For example, if you said that four stamps were used and that the postage was 21¢, this is not possible given the facts of the problem.]

Your postage problem

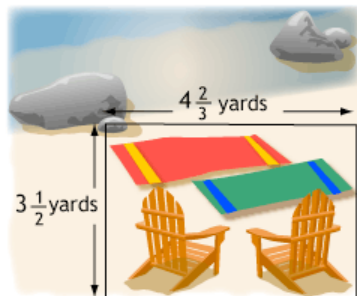
Solution to your postage problem

5. Use the following questions to help you write a paragraph reflecting on what you've accomplished in the unit so far:

- What have you learned about solving problems?
- What math concepts have you used? What new math skill or strategy did you develop?
- How did working with a partner help you?
- Which of the items in the Criteria for Quality Partner Work list did you and your partner use?
- What is one thing that you learned from your partner?

How can working with your classmates each day to solve problems help you expand your own set of problem-solving strategies and ideas?

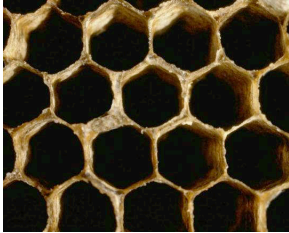
STAYING SHARP 1.4

PRACTICING ALGEBRA SKILLS & CONCEPTS	<div>1. Solve the shape equation puzzle:</div> <div><div><div>● + (▲ + ■) = 13</div><div>(▲ + ■) = 8</div><div>● + ▲ = 7</div></div><div><div>● = _____</div><div>▲ = _____</div><div>■ = _____</div></div></div>	<div>2. Find the sum of the digits of each multiple of 9 below.</div> <table><tr><td></td><td>Sum of digits</td></tr><tr><td>9 · 1 = 9</td><td>9</td></tr><tr><td>9 · 2 = 18</td><td>1 + 8 = 9</td></tr><tr><td>9 · 3 = 27</td><td></td></tr><tr><td>9 · 4 = 36</td><td></td></tr><tr><td>9 · 5 = 45</td><td></td></tr><tr><td>9 · 6 = 54</td><td></td></tr><tr><td>9 · 7 = 63</td><td></td></tr><tr><td>9 · 8 = 72</td><td></td></tr><tr><td>9 · 9 = 81</td><td></td></tr></table> <div>Describe a pattern. (Do you think it will continue?)</div>		Sum of digits	9 · 1 = 9	9	9 · 2 = 18	1 + 8 = 9	9 · 3 = 27		9 · 4 = 36		9 · 5 = 45		9 · 6 = 54		9 · 7 = 63		9 · 8 = 72		9 · 9 = 81	
	Sum of digits																					
9 · 1 = 9	9																					
9 · 2 = 18	1 + 8 = 9																					
9 · 3 = 27																						
9 · 4 = 36																						
9 · 5 = 45																						
9 · 6 = 54																						
9 · 7 = 63																						
9 · 8 = 72																						
9 · 9 = 81																						
PREPARING FOR UPCOMING LESSONS	<div>3. When full, the gas tank of a car holds 15 gallons. It now contains 12 gallons. What percent represents how full the tank is?</div> <div>Answer with supporting work:</div>	<div>4. Kathi starts with a number. She adds 3 to her number, then doubles the result. Finally, she subtracts 7. If she ended with 9, what number did she start with?</div> <div>Answer with supporting work:</div>																				
REVIEWING PRE-ALGEBRA IDEAS	<div>5. After selling tickets to a dance for \$3.25 each, the student council brought in \$1020.50 from ticket sales. How many tickets did the student council sell?</div> <div>Answer with supporting work:</div>	<div>6. Angela and Tina are at the beach. How long of a piece of rope do the girls need to totally enclose their space?</div> <div></div>																				

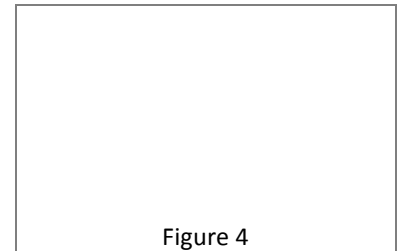
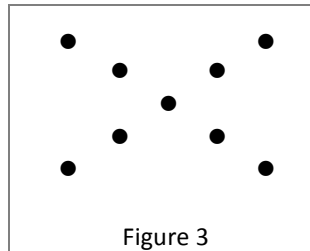
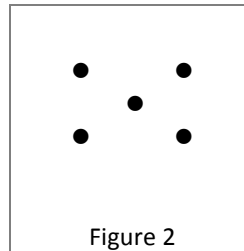
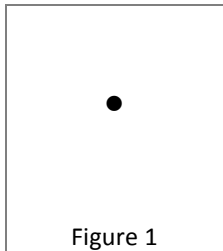
Lesson 1.5 Problem solving with patterns

1.5 OPENER

1. A honeycomb is an example of a pattern in nature. Write down two other patterns from nature or your everyday life.



2. Draw the next figure in the pattern shown:




3. Write the next three numbers in the following pattern: 1, 4, 9, 16, ____, ____, ____

1.5 CORE ACTIVITY

1. Many problems in mathematics can be solved by studying patterns. Explore the following examples with your partner to see whether you can identify any patterns.

- a. For each of the problems, if there is a pattern, check "P"; if there is no pattern, check "NP."

A.  P ☐ NP ☐

B. 2, 5, 8, 11... P ☐ NP ☐

C. 3, 6, 10, 15... P ☐ NP ☐

D.

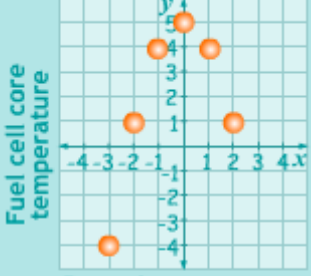
Fuel cells	1	2	3	4	5
Light years	2.5	5	7.5	10	12.5

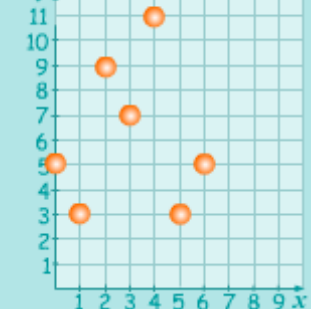
 P ☐ NP ☐

E.

Term number	1	2	3	4	5
Value of term	3	5	8	7	10

 P ☐ NP ☐

F.  P ☐ NP ☐

G.  P ☐ NP ☐

b. A **sequence** is a set of numbers that follows a pattern. A **term** is a value within a sequence. For the problems for which you identified a pattern, write the next 3 terms in the space below.

Problem (list by letter)	Next 3 terms

c. Share your answers with your partner. Take turns explaining to each other how you found the next terms in the pattern.

2. Here are three different sequences that start with the terms 2 and 6.



2, 6, 10, 14, ...	2, 6, 18, 54, ...	2, 6, 8, 14, 22, ...
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a. Write the next three terms for each sequence.


2, 6, 10, 14, _____, _____, _____	2, 6, 18, 54, _____, _____, _____	2, 6, 8, 14, 22, _____, _____, _____
-----------------------------------	-----------------------------------	--------------------------------------


b. Explain the pattern for each sequence.

2, 6, 10, 14, ...	2, 6, 18, 54, ...	2, 6, 8, 14, 22, ...
Explanation of pattern: 	Explanation of pattern: 	Explanation of pattern:

3. Sometimes, sequences can be found within a “growing shape pattern.”
- a. Use the shape pattern below to complete the table. [Hint: Look at the line segments of the following length, , around the sides of the figure and inside the figure.]
- 






- | | | | | |
|--|----------|----------|----------|----------|
| | Shape #1 | Shape #2 | Shape #3 | Shape #4 |
| Number of total line segments in pattern | | | | |
- b. Search for a pattern in the table. Based on this pattern, how many line segments will there be in Shape #10?
4. In the space below create your own sequence problem, but do not write the answer. Your problem can be either a pictorial sequence or a numerical sequence. You will show your problem to your partner; he or she will try to write or draw the next three terms in the sequence. Be sure to discuss your solutions.

5. The table below is an example of a tool called a math journal. You will use this tool to help you organize information about important math vocabulary and concepts. Although you can refer to your notes and work in the activity book, be sure to state ideas in your own words.

Vocabulary term	My understanding of what the term means	An example that shows the meaning of the term
a. Sequence		
b. Term		

1.5 CONSOLIDATION ACTIVITY

In order for you to be successful in mathematics, it is important for you to understand yourself as a learner of mathematics. When you have completed all four, trade papers back with your partner and discuss all four problems.

HOMEWORK 1.5

Notes or additional instructions based on whole-class discussion of homework assignment:

1. For each of the following sequences, write the next three terms; then explain the pattern.

Sequence	Next three terms			Explanation of pattern
a. 1, 3, 5, 7, 9, ...				
b. 0, 2, 1, 3, 2, 4, 3, ...				
c. 17, 12, 7, 2, -3, ...				
d. 8, 4, 2, 1, $\frac{1}{2}$, ...				
e. -35, -28, -21, -14, ...				
f. 1, 2, 6, 24, 120, ...				

2. For the “growing dot” sequence shown,

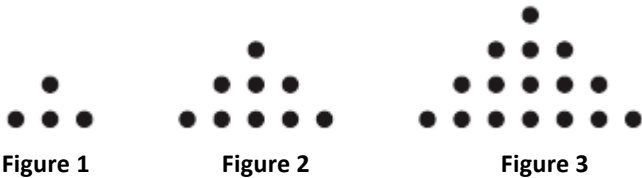


Figure 4

Figure 5

- a. Draw the next two figures in the pattern.
- b. Starting with Figure 1 and using the number of dots in each figure, list the first five numbers in the sequence.
- c. Describe the pattern in the sequence.
- d. Use your pattern to determine the number of dots in the 8th figure in the pattern.

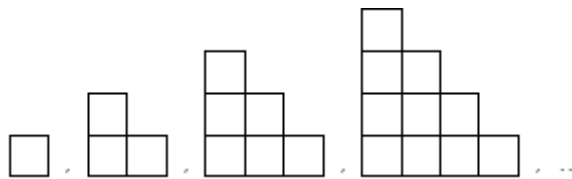
3. Write the next three terms in each of the following sequences. Then explain the pattern.

a. $-19, -15, -11, -7, \dots$

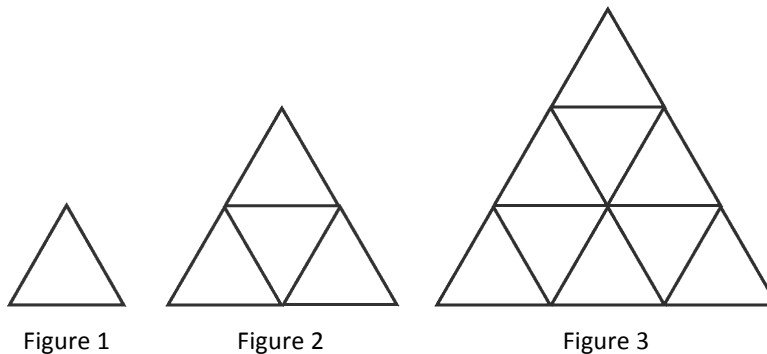
b. $-4, -8, -16, -32, \dots$

c. $5, 7, 11, 17, 25, \dots$

4. If the pattern of blocks were to continue, how many blocks would be in the eighth figure? Explain how you know your answer is correct.



5. A picture pattern is shown below. In Figure 2, there are a total of 4 triangles of the same size; 3 of the triangles “point up” and 1 triangle “points down”.



- For Figure 7, how many total triangles will there be?
- For Figure 7, how many triangles will there be that “point up”?
- For Figure 7, how many triangles will there be that point down?

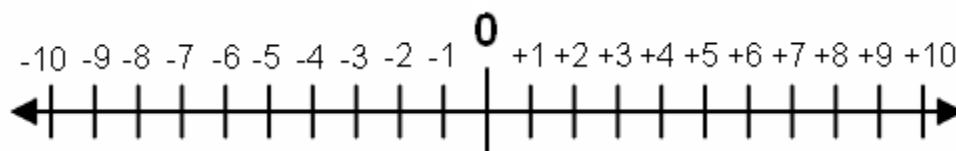
6. A grocery store owner is planning a display by stacking cans of soup in a pyramid. The top row of the stack will have 1 can, the row underneath it will have 3 cans, the row underneath it will have 5 cans, and so on. If the owner wants the display to have a total of 12 rows, then how many cans will he need? Show your work. Explain how you know your answer is correct.

STAYING SHARP 1.5

PRACTICING ALGEBRA SKILLS & CONCEPTS	<div>1. Solve the shape equation puzzle:</div> <div><div><div>▲</div><div>+</div><div>▲</div><div>+</div><div>◆</div><div>=</div><div>21</div></div><div><div>◆</div><div>+</div><div>▲</div><div>=</div><div>16</div></div></div> <div><div><div>▲</div><div>=</div><div>_____</div></div><div><div>◆</div><div>=</div><div>_____</div></div></div>	<div>2. Continue the pattern in the table below.</div> <table><tr><th>Step</th><th>Value</th></tr><tr><td>1</td><td>3</td></tr><tr><td>2</td><td>7</td></tr><tr><td>3</td><td>11</td></tr><tr><td>4</td><td></td></tr><tr><td>5</td><td></td></tr><tr><td>6</td><td></td></tr><tr><td>7</td><td></td></tr></table> <div>Describe the pattern and how you continued it.</div>	Step	Value	1	3	2	7	3	11	4		5		6		7	
Step	Value																	
1	3																	
2	7																	
3	11																	
4																		
5																		
6																		
7																		
PREPARING FOR UPCOMING LESSONS	<div>3. On a map of Texas, the distance from Abilene to Midland is 10 cm. The map’s scale shows that 4 cm on the map equals 50 miles in real life. What is the actual distance from Abilene to Midland in miles?</div> <div>Answer with supporting work:</div>	<div>4. There are 6 boys to every 9 girls in the 6th grade at Ripple River School. There are a total of 75 students in the 6th grade. How many boys are there in the 6th grade?</div> <div>Answer with supporting work:</div>																
REVIEWING PRE-ALGEBRA IDEAS	<div>5. Order these decimals from least to greatest:</div> <div>7, 0.62, 0.38, 0.9, 0.54, 0.6, 0.27, 0.106, 0.003</div> <div>Answer with supporting work:</div>	<div>6. A rectangle has a length of $3\frac{1}{5}$ yards and a width of $7\frac{5}{6}$ yards. Make a sketch of the rectangle and label its dimensions.</div> <div>Estimate the area of the rectangle.</div> <div>Calculate the area of the rectangle.</div>																

Lesson 1.6 Problem solving with patterns—continued

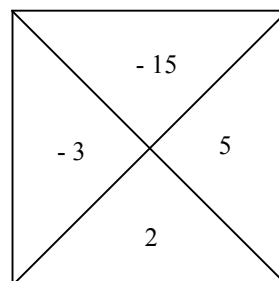
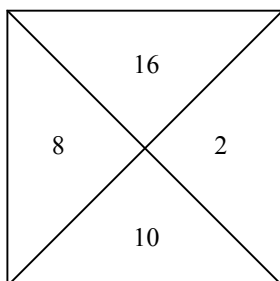
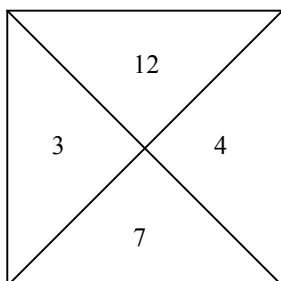
1.6 OPENER



1. Locate all of the numbers on the number line that are 1 unit away from zero.
 - a. What do you notice about these numbers?
 - b. What happens if you add these numbers?
2. Locate all of the numbers on the number line that are 3 units away from zero.
 - a. What do you notice about these numbers?
 - b. What happens if you add these numbers?

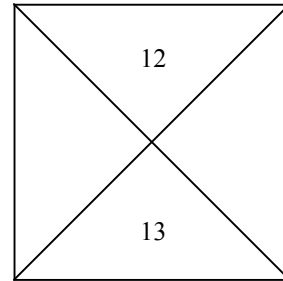
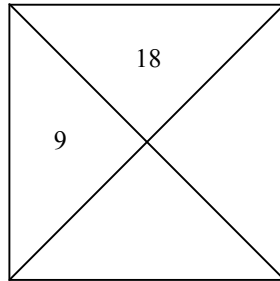
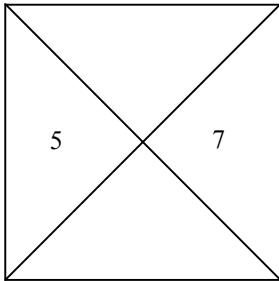
1.6 CORE ACTIVITY

Study the numbers in the different parts of the box and look for relationships.

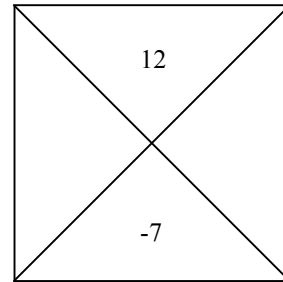
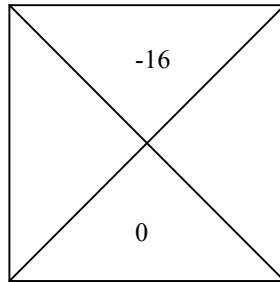
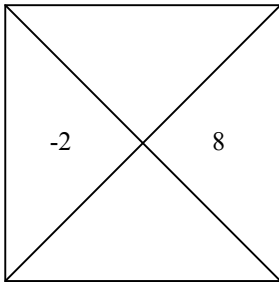


1. What is the relationship between the two numbers in the left and right parts of the box and the number at the top of the box? Explain in a full sentence.
2. What is the relationship between the two numbers in the left and right parts of the box and the number at the bottom of the box? Explain in a full sentence.

3. Use the pattern that you discovered to complete the following square box problems.



4. The following square box problems involve some negative numbers. Use the pattern that you discovered to complete these square box problems.



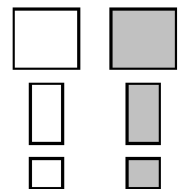
1.6 CONSOLIDATION ACTIVITY

1. Name a math skill for which you have automaticity.
2. Name a math skill for which you do not yet have automaticity and need some more practice.



Complete the following problems.

3. Label the algebra tiles. (Shaded tiles represent negative quantities.)
4. Which tiles represent positive and negative integers?



5. Build the following problems using algebra tiles, cross off zero pairs (if they exist), and give the final answer.

a.

-3	+	3	=	

b.

-5	+	2	=	

c.

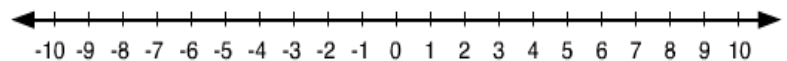
-2	+	7	=	

d.

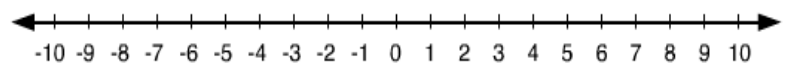
-5	+	-2	=	

6. Model the following problems using a number line and record your answer. The arrow's direction indicates the sign, and the arrow's length represents the size of the number.

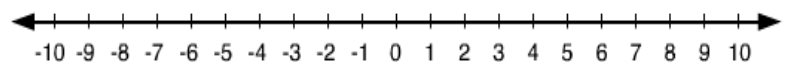
a. $-3 + 3 =$



b. $-5 + 2 =$

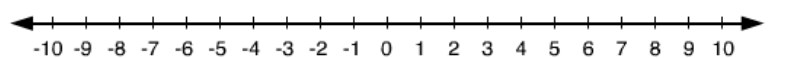


c. $-2 + 7 =$



- d. Rewrite $-5 - 2$ as an addition problem and find the answer.

$-5 - 2 =$



7. Model the following problems using either algebra tiles or a number line and record your answer.

a. $4 + 3 =$

b. $-8 + 5 =$

c. $-2 + 7 =$

d. $-1 - 6 =$

8. What patterns have you noticed? State conclusions for adding and subtracting signed numbers:

a. When subtracting signed numbers, change the problem into _____.

b. When adding signed numbers:

i. If both numbers are positive, then the sum is _____.

ii. If both numbers are negative, then the sum is _____.

iii. If one number is positive and one number is negative, then the sum can be found by _____.

[Hint: Use the idea of “absolute value” in your answer.]

HOMEWORK 1.6

Notes or additional instructions based on whole-class discussion of homework assignment:

1. Change the following subtraction problems into addition problems.

a. $-2 - 3$

b. $8 - (-1)$

c. $-4 - 9$

d. $-1 - (-2)$

e. $(-6) - 2$

2. Model the following problems by drawing algebra tiles and crossing off zero pairs (if they exist). Report a final answer.
[If necessary, change subtraction problems into addition problems as a first step].

a. $5 + (-2)$

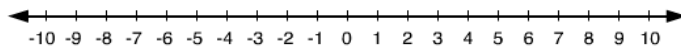
b. $-2 + 4$

c. $3 - (-4)$

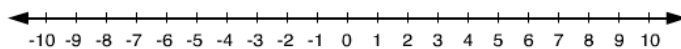
d. $-1 - (-5)$

3. Model the following problems using a number line and record your answer. Remember that the arrow's direction indicates the sign, and the arrow's length represents the size of the number.

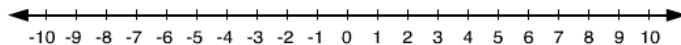
a. $-4 + 6 =$



b. $8 + -3 =$

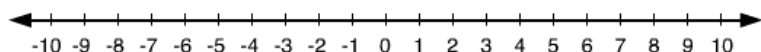


c. $-2 + -7 =$



4. Rewrite $-6 - (-4)$ as an addition problem and give the final answer. Model the expression using a number line.

$-6 - (-4) =$



5. Solve the following problems. Make use of the fact that subtracting a number is the same as adding its opposite. If needed, use either the algebra tile model or the number line model to help you.

a. $-20 + 8 =$

b. $-17 + 5 =$

c. $10 - (-10) =$

4. $7 + -5 =$

d. $-16 + -17 =$

e. $6 - (-4) =$

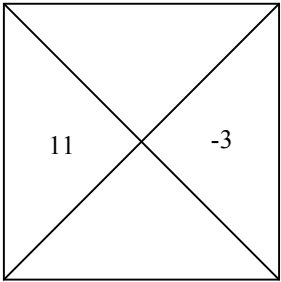
f. $-5 - (-20) =$

g. $1 - 13 =$

h. $13 + -17 =$

i. $-20 - 14 =$

STAYING SHARP 1.6

PRACTICING ALGEBRA SKILLS & CONCEPTS	<p>1. Solve the shape equation puzzle:</p> $\blacktriangle + \blacklozenge = \blacktriangle$ $\blacksquare + \blacklozenge = 6$ $\blacktriangle + \blacksquare = 13$ $\blacktriangle = \underline{\hspace{2cm}}$ $\blacklozenge = \underline{\hspace{2cm}}$ $\blacksquare = \underline{\hspace{2cm}}$	<p>2. In a Square Box Problem, the:</p> <ul style="list-style-type: none"> top number is the product of those at left and right bottom number is the sum of those at left and right <p>Complete this Square Box Problem.</p> 
PREPARING FOR UPCOMING LESSONS	<p>3. The amount of protein a person needs each day is proportional to his or her weight. Rod weighs 165 pounds and needs 60 grams of protein each day. If Todd weighs 220 pounds, how much protein does he need each day?</p> <p>Answer with supporting work:</p>	<p>4. Sketch a rectangle and label its dimensions (that is, write the numbers for its length and width) that meets the following conditions:</p> <ul style="list-style-type: none"> Perimeter = 18 centimeters Area = 20 square centimeters <p>How do you know the rectangle meets both conditions?</p>
REVIEWING PRE-ALGEBRA IDEAS	<p>5. Round each decimal number to the nearest integer:</p> <p>11.90</p> <p>5.49</p> <p>7.09</p> <p>8.5</p> <p>-2.2</p>	<p>6. You can find the volume of a rectangular solid using the formula Volume = length · width · height, or $V = lwh$.</p> <p>Find the volume of the rectangular solid pictured here.</p> 